METAPHYSICS WITHIN STRING PHYSICS

Modal realism after String Theory

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This talk is a multilayer presentation. It unravels a landscape of interconnected ideas. The rationale behind this choice has been somehow imposed by the multidisciplinary nature of my work during the last few years.

The presentation is based on some parts of my forthcoming book (“Emergent Spacetime in String Theory" - Routledge, 2017).

It is also a work in progress for an invited contribution to a miscellaneous volume - “Beyond Spacetime: The Philosophical Foundations of Quantum Gravity" - submitted to Cambridge University Press.

This presentation mainly divides in two parts.
(1) Background independence of string theory explored through the theory's moduli space - an argument formulated in the mathematical language of deformation theory.
(2) The formal articulation of the argument turns out to be useful to my attempt of naturalizing the metaphysics of possible world.

The hope is the following: on the one side promoting a form of *inductive metaphysics*, namely a metaphysics sensitive to the "empirical sources" imported from physics; on the other side letting this type of metaphysical insights add some important dimensions to the technical debate within the physics circles.

What do I mean here with “empirical sources"?
Specifically, empirical sources here amount to be the fundamental physical scenario (maybe not the most fundamental one)) delivered by string theory. That is, the fundamental physical ontology of string theory.

Why does the physical ontology of string theory count as “empirical sources"?

- Risk of empirical incoherence (Maudlin 2009)

- A way of rescuing the theory from the charge of being empirical incoherent (Huggett, Wütrich, 2013), along with the claim that string theory is also empirically adequate, (Huggett, Vistarini, 2015), (Dawid, 2013), and (Vistarini, 2017).

As I have been working on string theory background independence I discovered some philosophically interesting features of the theory: if one digs into the mathematical and physical aspects of that background independence, one may find out that the theory's account of spacetime emergence is a quite **philosophically conservative** one: it **smoothly revises** or **extends** the traditional notion of mechanistic explanation. Not a straightforward achievement, if we think to the historical use of the notion of mechanistic explanation.

Why **smoothly**? There is a meaningful sense in which we can say that already in any classical, pre-relativistic hamiltonian we can trace an embryo of spacetime emergence - see (Harvey, 2005); (Albert 2016); (Vistarini,2017).
And both general relativistic spacetime emergence and emergence of the extra dimensions (through dualities arguments) in string theory develop that notion in a more radical physical scenario - (Vistarini 2017).

Suppose you have a pre-relativistic, classical scenario populated by, say, N particles in a three-dimensional Euclidean space. Its hamiltonian looks like:

$$\sum_{i=1,..N}[...]+\sum_{K=1,..,N-1}V_{kj}((x_k-x_j)^2+...+(z_k-z_j)^2).$$

$k \neq j$ The x,y and z are cartesian coordinates in the three dimensional space, here appearing to be equipped with an Euclidean metric.

Further simplification: let's pretend that all we need to account for ordinary macroscopic systems (clocks, rods, cats and so on) is the set of interactions described by the classical potential V - that is, let's pretend we are inhabitants of a world which is fundamentally classical and in which there is no such a thing like quantum behaviours.

How would a world like this *appear* to us? Here the expression “world’s appearances" does not contain any reference to our subjective experience. Rather, it simply denotes what the world looks like through the outcomes of experiments and of measurements.
A physical world like this would appear, without any doubt, to be three-dimensional and Euclidean.

Why is that? The answer is given by the structure of the potential V. The latter is an explicit function of the three dimensional Euclidean distances. And any potential dictates how any material interaction occurs.
One may say that these interactions make the Euclidean distances manifest.

That is, the potential produces a **manifest** geometry of the world.
In general and not only in this toy case, our access to any presumed actual geometry can only be achieved by reading into the Hamiltonian.
But what if things are not so simple? What if the manifest geometry and the presumed actual one do not coincide?

There is an old story about this possible geometrical mismatch. It is the parable by Poincairé about the difficulty of establishing beyond any doubt the status of geometrical knowledge. According to him the issue cannot be settled by means of empirical evidence. Poincairé was using this parable to debate about epistemology of geometry. Here I draw a lesson at right angle with all that.

The Poincairé's story is about an imaginary two dimensional world equipped with Euclidean geometry. It is a disk, the one that contrives by means of effects of spatial variation of the temperature on the lengths of measuring rods.

The inhabitants, unaware of the hidden dynamics affecting rods' lengths, get out of measurements and of dynamical generalization a manifest geometrical image of the world that is not the same as the actual one.

So they conclude to be living in a infinite Lobachevskian plan.
One may say in this case the dynamics are producing a manifest geometry of the world that does not coincide with the actual one.

What this parable suggests to me is that we may generalize the classical scenario just described to a case of arbitrarily curved background. In this case the Hamiltonian would depend on generalized coordinates x,y and z. In a world like this there is no general and unique way of defining the line element ds^2, that is, there is no uniform way of defining the distance between particles.

Nonetheless, in a hamiltonian like this, the potential can appear (by means of local coordinates transformations) to be function of $(\delta s)^2 = (\delta x)^2 + (\delta y)^2 + (\delta z)^2$:

$$\sum_{i=1,..N} [...] + \sum_{K=1,..,N-1} V_{kj}((\delta x)^2 + (\delta y)^2 + (\delta z)^2).$$

$k \neq j$.

That is, interactions between particles can be seen as manifesting an Euclidean flat space geometry. And so this world now in spite of the fact that it is actually curved, it appears to to be flat and globally Euclidean.

So, we might wonder, what does it means to say that space is actually curved in this case?
Indeed, the actual curved geometry of the background appears to have no role in the production of the flat geometrical appearances. **It is the dynamics that do all the work**.
So, one may take a further step and conclude that the hamiltonian produces a manifest geometry of the world independently of **whether or not there is an actual background geometry at all**.

From this reformulation of the Poincairé's under-determination problem we gain a lesson: still remaining in the classical scenario, there is a legitimate way of reading the physics according to which space and time are on the side of what is mechanically explained, rather than being on the side of those physical features somehow prior to physical processes.

As long as we remain confined to classical physics, reading dynamics in one way or the other is a matter of choice. But as we move to the quantum scenario, the reading of emergence turns out to be more truthful.

My argument in favour of string theory background independence uses a line of reasoning mimicking the same logic, but with an important distinction. As soon as we generalize the story to a quantum hamiltonian the manifest image of the world stops being a competitor of the presumed actual one (like in the original Poincairé formulation) since they now belong to two distinct levels of description of reality governed by different physical parameters.

The main claim is that string theory does not posit any fundamental spacetime geometry. Spacetime is a mechanical byproduct of underlying dynamics. This seems to hold true for both formulations of the theory: perturbative and non-perturbative ones - the former gained by perturbating the classical action and by reimposing conformal invariance for the quantum string action, the latter delivered by the AdS/CFT duality.

The perturbative formulation - at least in my reading - admits general relativistic spacetime in the sense mentioned above. The theory does not posit any fundamental geometry and general relativistic spacetime is a mechanical byproduct of underlying strings dynamics.
Some important mathematical/physical results:(Polchinski,2001);(Witten, 1997);
On the philosophical interpretation of the derivation of General Relativity from string theory: (Huggett, Vistarini, 2015);(Vistarini, 2017, forthcoming).

Also, in the non-perturbative case, and in all those cases in which arguments via duality are involved (T-duality, mirror symmetries) my reinterpretation of Poincairé problem points to the fact that any posit regarding fundamental geometry in the theory (in this case also involving geometry of the extra dimensions) does not play any explanatory power.

$$
\begin{array}{l}
S \times K \\
\downarrow^{\iota} \\
S
\end{array}
$$

Moduli spaces are abstract spaces used to parameterize families of objects. Here our families of objects are families of spacetimes (equipped with metric tensors).
What does it mean “spaces of parameters"?
A moduli space M is a space of parameters for some family K of physical spacetimes *only if* the correspondence between ”points" of M and spacetimes in K is *well-defined*.

Well-defined in some basic mathematical sense means something like:

$$\begin{array}{c} K \\ \downarrow^{\phi} \\ M \end{array}$$

each object in K is mapped on a unique point of M, but this is not required in the other way around, that is $Ker(\phi)$ is not required to be equal to $\emptyset$. Now, this requirement by itself would only produce some humble structure. Better if we require some more constraints.

In order to avoid some humble structure a second requirement may be that whenever two objects in K are somehow "similar" or "close" with respect to some property, the corresponding "points" on M must be close as well with respect to some M's topology - which in principle is not the same as that in K.
This constraint induces a refinement on the previous one about having a well-defined map: things in K that get mapped onto the same point in M, must be at least quite "similar".

All this is a toy version of how you might get a "fine" moduli space - the one encoding as much rich information as possible.
This moduli space should possibly encode intra-relations among parts of a single object, if any, and also trans-objects (or inter-objects) relations among objects in the family.
In this context objects are spacetimes.

It turnes out that the “local" topological structure of this simplified version of the theory moduli space, along with some fiber bundle structure (conveying dynamical information) on top oreveal the genuine metaphysical commitment of string theory to the non fundamentality of spacetime. But it also reveal that the fundamental ontology of string theory contains a philosophical notion of *possibilia* that might be seen as a naturalized version of the purely metaphysical notion delivered by Lewis modal realism.

$$\begin{array}{ccc} K & & \\ \downarrow^{\phi} & & \\ B & \subseteq & M \end{array}$$

where

$$K = \bigcup_{\lambda \in B} K_\lambda, \tag{1}$$

$\forall\ \lambda \in B$, $\phi^{-1}(\lambda) = K_\lambda$ -(Kodaira, 2005).
B for simplicity is here a one-dimensional set of parameters. Moreover each K_λ is an individual spacetime structure. Let's just focus on the compact part of the higher dimensional space. Here by heavily relying on (Kodaira, 2005), each K_λ is a compact, complex manifold. B is a complex domain.

(Kodaira 2005) Suppose given a domain B in and a set $\{K_\lambda | \lambda \in B\}$ of complex manifolds K_λ depending on λ. We can say that K_λ will have a C^∞ dependence on λ and that $\{K_\lambda | \lambda \in B\}$ is a *differentiable family* of compact complex manifolds, if there are a differentiable manifold K as above and a C^∞ map ϕ of K onto B satisfying the following conditions:

(1)The differential of ϕ, ϕ_*: $T_p K \longrightarrow T_{\phi(p)} B$ is surjective at every point $p \in K$, where $T_p K$ and $T_{\phi(p)} B$ are respectively the tangent space to K at the point p and the tangent space to B at the point $\phi(p)$.

(2) K is a non empty complex compact manifold and for each $\lambda \in B$, $\phi^{-1}(\lambda) = K_\lambda$ is a compact differentiable submanifold of K.

(3) There are locally finite open covering $\{V_j | j = 1, 2, ...\}$ of K and complex-valued C^∞ functions $z_j^1(p), ..., z_j^n(p)$, $j = 1, 2, ...$, defined on V_j such that for each λ the following coordinatization form a system of local complex coordinates of K_λ:

$$\{p \rightarrow (z_j^1(p), ..., z_j^n(p)) | V_j \bigcap \phi^{-1}(\lambda) \neq \emptyset\},$$

We are now confined to a case in which deforming the gluing functions $f_{jk}(z_k, \lambda)$ of a compact complex manifold means deforming its complex structure. Although all the K_λs of the family, along with K_0, share the same topological structure (or stronger, same differentiable structure), each fiber of the family has its own distinct complex structure. Generally a complex structure over a manifold (whether or not compact) comes along with a hermitian metric compatible with it, which in principle is not unique.

There is no a priori fixed relation between metric and complex structure, rather many different compatibility conditions, namely, many different mathematical correspondences involving holomorphic functions in the manifold's atlas of charts. I won't give here detail on such mathematical relations. It suffices to say that given a compatibility condition (Kodaira, 2005) between complex structure and an induced metric, any deformation of the complex structure is also a deformation of the metrical one.

Therefore, we have a family of geometrically inequivalent, topologically equivalent backgrounds. Now, the differentiable structure of any individual fiber in the family is not the same as the "differentiable" structure over the moduli space. The latter is a topological structure technically arising from the C^∞ act of deformations of one fiber into another. Within this type of family of deformations, we now pick a specific subtype, namely, first-order infinitesimal deformations.

Deforming the gluing functions by means of first-order derivation basically means:

$$\frac{\partial f_{ik}}{\partial \lambda} = \frac{\partial f_{ij}}{\partial f_{jk}} \frac{\partial f_{jk}}{\partial \lambda}. \tag{2}$$

The derivative of the gluing functions gives some "rate of geometrical change" with respect to the parameter λ of the family. The derivative gives information on all the directions along which the original metric structure may bend when deformed.

The rationale here is still finding a concise, still general way to express that if we suitably change the geometrical structure of some provisionally posited background physical observables of the system do not change their expecation values.

Now, by skipping a huge amount of detail, if we take the derivative of any gluing function over an infinitesimal disk, we produce, morally speaking, a holomorphic field at $\lambda = 0$, namely,

$$\theta_{ik}(0) = \frac{\partial f_{ik}(z_k, \lambda)}{\partial \lambda}|_{\lambda=0}. \quad (3)$$

It turns out that $\theta_{jk}(0)$ is a 1-cocycle of the sheaf T_{K_0}, namely, the sheaf of holomorphic vector fields over K_0. Its cohomology class $\theta(0)$ is an element of the cohomology group $H^1(K_0, T_{K_0})$.

But what does it mean? Informally speaking, the cohomology group $H^1(K_0, T_{K_0})$ is simply a vector space, whose vectors (cohomology classes) can be read in more familiar terms as closed and non-exact differential 1-form.
(In differential geometry, a one-form on a differentiable manifold is a smooth section of the cotangent bundle).

The cohomology group $H^1(K_0, T_{K_0})$, generated as vector space by a basis of non isomorphic θs, represents all the infinitesimal deformations of K_0. It appears in the Kodaira-Spencer map:

$$\rho : T_{B,0} \longrightarrow H^1(K_0, T_{K_0}).$$

Locality on the moduli space is not locality in the ordinary sense (things are close in spacetime). Rather locality has to do with the degree of similarity among different spacetimes.

Still on a conceptual note, I want to make two points. First, the simplified version of moduli space is a mathematical structure depicting a space of mathematical possibilities that here is confined to the formal language of smooth deformation theory applied to compact complex manifolds. Things can be more complex than this if the action of deformation is not a smooth one.

Also,by taking in consideration any smooth deformation of some geometry, it is not always the case that one can find some physical geometry admitted in the theory. Physical possibilities are a subset of the mathematical ones.
In the part concerning the revision of modal realism, mathematical possibilities of this sort replace purely logical possibilities - the latter traditionally inspired by ordinary intuitions about how things might be different from what they are.

Second, the existence of a Kodaira-Spencer map at $\lambda = 0$ means that each tangent vector at $\lambda = 0$ to the moduli space identifies a possible deformation of the geometry K_0.
If the Kodaira-Spencer map is surjective, then every first order deformation of a geometry K_0 is represented by a tangent vector to the moduli space.

The condition under which the Kodaira-Spencer map is surjective are beyond the goal of this talk. However, if the map is surjective, the family of first order deformations K is *complete*, namely, it contains all the possible first-order deformations of K_0.
So, under the condition of suriectivity of the Kodaira-Spencer map we gain a an exhaustive landscape of mathematical possibilities.

$$\begin{array}{c} \overline{H} \\ \downarrow^{p} \\ M \end{array}$$

The fiber bundle $\overline{H}$ is defined as $\sqcup_{\lambda \in M}(H_\lambda \times C)$. Each disjunct $H_\lambda \times C$ is a fiber over a point λ of the moduli space M. Each λ parameterizes some spacetime structure. H_λ is the Hilbert space of the states of the system Q whose dynamics unfold along the spacetime parameterized by λ and the vector space of complex numbers C represent all the numerical values assumed over λ by the transition functions $f_{\alpha_1 \alpha_2 \ldots \alpha_n}$ of the system.

A claim of background independence with respect to spacetime geometry may be the following: **any local family of different spacetime geometries, still topologically equivalent,can be taken as the data for constructing a string theory without any prejudice to choice of a particular member.**.

That is, for any family of topologically equivalent fibers given a set of observables and an arbitrarily picked member λ_0 of the family, the value $f_{\alpha_1 \ldots \alpha_n}(\lambda_0) = < O_{\alpha_1} O_{\alpha_2} \ldots O_{\alpha_n} >$ remains constant over the family, i.e. $\forall \ \lambda \in M$

$$f_{\alpha_1 \ldots \alpha_n}(\lambda_0) = f_{\alpha_1 \ldots \alpha_n}(\lambda).$$

T-duality of the bosonic string is a toy example of this strings physics blindness to geometry.

A claim of background independence with respect to the topological structure may be the following: **any local family of topologically inequivalent spacetimes can be taken as the data for constructing a string theory without any prejudice to choice of a particular member**. Mirror symmetries not preserving topological invariance.

PART 2

It is held into the philosophy of quantum gravity circles that endorsing Lewis ontology of modal realism is incompatible with endorsing the fundamental physical ontology of any quantum gravity theory.

I argue that this apparent incompatibility can be bypassed as long as modal realism is revised in "naturalistic terms". It might turn out that this revision, if made in light of string theory formal articulation and physical ontology, can produce a metaphysics of possible worlds compatible with the non-fundamentality of space and time.

A first step toward this "naturalization" of the system may be that of replacing a notion of logical possibilities mainly guided by ordinary language and ordinary intuitions with a notion of mathematical possibilities imported from the formal articulation of string theory.

FEW INTRODUCTORY REMARKS:

(1) Lewis modal realism thesis claims that this world is just one among other more or less similar to it.

A proposition p is possibly true if and only if p is true in one of this worlds. A proposition is necessary true if and only if it is true in every possible world.

In relation to that, Lewis thesis of modal realism held that what it takes for a proposition like "you are sad" to be true in another world is not for you to be sad in that world since you are not there. Rather it is for your counterpart to be sad in that world.

(2) "I believe, and so do you, that things could have been different in countless ways. But what does this mean? Ordinary language permits the paraphrase: there are many ways things could have been besides the way they actually are. I believe that things could have been different in countless ways; [...] I therefore believe in the existence of entities that might be called ways things could have been. I prefer to call them possible worlds." (1973a: 84)

(3) Lewis believed that modality cannot make sense without the metaphysical assumption of modal realism. He thought that we cannot determine that " F is possible" without an idea of what a real world where F holds would look like. In deciding whether it is possible for "squirrels to be inside atoms" we do not simply determine whether the proposition is grammatically coherent, we actually think about whether a real world would accommodate such a state of affairs. Thus we require modal realism if we want to use modality at all.

(4) This metaphysical setting turns out to be useful in the analysis of counterfactuals, which in turn are central to an analysis of causation. A trivial example of how causation and counterfactual can be related to each other:
as I touch a certain key on my computer, a slides sequence appears in front of your eyes. Should I touch a different key, a different sequence would appear in front of you. That is how the actual order of the slides you see causally depends on the keystroke (Lewis, 1986)

Two properties of modal realism here revised:
Closeness: Lewis's theory of counterfactuals relies on comparisons between possible worlds. It relies on comparisons between this world and other worlds.
Properties: The modal realist takes properties to be sets of possibilia.

Lewis (1987)
What does it make a collection of things - the one which is the least inclusive - a Lewis possible world: mainly the fact that the parts of the collections are unified by spatio-temporal relations: “whenever two possible individuals are spatiotemporally related they are worldmates". The fact that the most satisfying identikit of a Lewis possible world must show its internal spatiotemporal unifying structure suggests that space and time are fundamental.

A first comment: “fundamental" in the Lewis sense of world-identification is not the same as “fundamental’ in the sense of quantum gravity. The latter is grounded (at least in the case of string theory) on a notion of physical length scale: fundamental physics seems to unfold below a certain threshold where ordinary notion of space and time break down.
The former instead does not explicitly refer to an idea of fundamentality in terms of quantum physical scale.

Lewis (1987)
As far as I understand, the fundamental role of space and time in Lewis has more to do with world-identification purposes.
And within this framework there aren't any explicit constraints ruling out that these worlds may be emergent.
Lewis idea of fundamentality of space and time does not seem to claim something more than their being real. This is not in tension with any quantum gravity scenario. No serious physical theory would deny that space and time are real.
The Lewis-pluriverse might well be an emergent one.

The line of reasoning about the different meanings of "fundamental" is an attempt to bypass a powerful objection against Lewis's approach made by Christian Wütrich (2015):“despite its modal *embarras de richesses*, Lewis pluriverse does not contain our world" (page 9).

A second comment is: Lewis (1986): page 70
“Trans-world comparisons, yes; trans-world spatiotemporal relations, no"
This statement concisely exemplify the notion of isolation. The only trans-world relation allowed by Lewis is that defined by a similarity order.
“So, things that are parts of the two worlds may be “simultaneous" or not, they may be in the same or different towns, they may be near or far from one another, in very natural counterpart theoretic sense. But these are not genuine spatiotemporal relation across worlds".

The “action" of deforming spatiotemporal structures produces a notion of locality over the string theory moduli space that is very different from the ordinary notion of locality in spacetime. "Points" are close there in virtue of the degree of topological similarity of the worlds they both parameterize. In this sense the mathematical act of deformation formally encodes the qualitative ordinary notion of isolation. It exemplifies a non spatiotemporal relation among worlds related by similarity order. In this sense it might be considered the rigourous mathematical counterpart of the Lewisian notion of trans-world isolation.

A third comment is about a big challenge against my attempt of rescuing the Lewis approach from rejection (in light of quantum gravity theoretical achievements): Humean supervenience.
Humean supervenience: there is nothing to reality except the spatio-temporal distribution of local natural properties. Laws supervene on these distribution.

String dualities might be considered a violation of Humean supervenience. String dualities shows stings dynamics insensitivity to different spatio-temporal structures, along with insensitivity to topologically differences.