Conceptual Issues in Quantum Cosmology

Claus Kiefer

Institut für Theoretische Physik
Universität zu Köln

Why quantum cosmology?

Gell-Mann and Hartle 1990:
Quantum mechanics is best and most fundamentally understood in the framework of quantum cosmology.

- A universally valid quantum theory must be applied to the Universe as a whole as the only closed quantum system in the strict sense;
- need quantum theory of gravity, since gravity dominates on large scales

Contents

The superposition principle

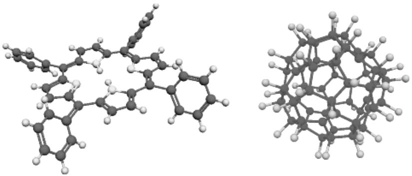
- Let Ψ_1 and Ψ_2 be physical states. Then, $\alpha\Psi_1 + \beta\Psi_2$ is again a physical state.
  For more than one degree of freedom, this leads to the entanglement between systems (*Verschränkung*).
- Linearity of the Schrödinger equation: the sum of two solutions is again a solution.

"Classical states" form only a tiny subset in the space of all possible states.

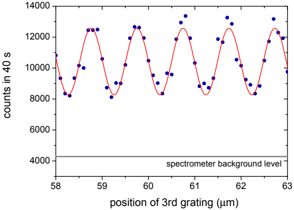
Erwin Schrödinger 1935:

I would not call that *one* but rather *the* characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought. By the interaction the two representatives (or ψ-functions) have become entangled. . . . Another way of expressing the peculiar situation is: the best possible knowledge of a *whole* does not necessarily include the best possible knowledge of all its *parts*, even though they may be entirely separated . . .

A particular example (Vienna experiment)



tetraphenylporphyrin ($C_{44}H_{30}N_4$) (left) and fluorofullerene $C_{60}F_{48}$ (right)



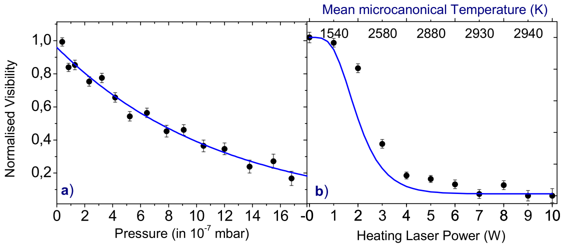
Interference pattern of tetraphenylporphyrin

L. Hackermüller *et al.*, *Phys. Rev. Lett.* **91** (2003) 090408

Decoherence

- Irreversible emergence of classical properties through the unavoidable interaction with the environment.
- Decoherence is based on an arrow of time
- Objects can then *appear* classically, although they are fundamentally described by quantum theory.
- First paper by Zeh (1970); important conceptual and quantitative developments in the early years by Zeh (1971, 1973), Kübler and Zeh (1973), Zurek (1981, 1982), Harris and Stodolsky (1981, 1982), Caldeira and Leggett (1983), Joos (1984), Joos and Zeh (1985), . . . ; experimental tests since 1996

Decoherence: Experimental test



Left: Decoherence through particle collisions.

Right: Decoherence through heating of fullerenes.

From: M. Arndt and K. Hornberger, Quantum interferometry with complex molecules, `arXiv:0903.1614v1`

What can be understood by decoherence?

- Classical properties are not an attribute of an isolated system; they are “defined” by the environment; importance of pointer states
- The decoherence time is tiny in macroscopic situations; this leads to the appearance of “events, particles, quantum jumps” (apparent collapse).
- Decoherence is experimentally well established (Cf. Nobel prizes for Haroche and Wineland 2012)

What cannot be understood by decoherence?

Is standard (unitary) quantum theory universally valid or not?

If yes, the Everett interpretation holds, with decoherence as an important ingredient. If not, an alternative theory (such as GRW-type collapse theories) must be seeked.

Important open questions:

- Why are there local observers?
- What is the origin of irreversibility?

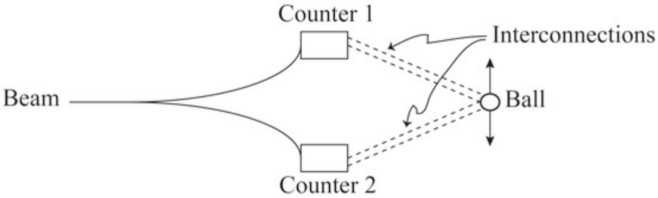
In the following:

- Situations with a (quantum) gravitational field
- Assume universality of superposition principle (i.e. no discussion of scenarios à la Diósi, Penrose, and others)

What about gravity?

Richard Feynman 1957:

. . . if you believe in quantum mechanics up to any level then you have to believe in gravitational quantization in order to describe this experiment. . . . It may turn out, since we've never done an experiment at this level, that it's not possible . . . that there is something the matter with our quantum mechanics when we have too much *action* in the system, or too much mass—or something. But that is the only way I can see which would keep you from the necessity of quantizing the gravitational field. It's a way that I don't want to propose. . . .



Gravitational cat states?

There are various suggestions to create a superposition of masses that only interact by their gravitational fields.

See e.g. Derakhshani, Anastopoulos, Hu, arXiv:1603.04430, or Marletto and Vedral, arXiv:1707.06036

Main approaches to quantum gravity

No question about quantum gravity is more difficult than the question, “What is the question?”
(John Wheeler 1984)

- Quantum general relativity
  - Covariant approaches (perturbation theory, path integrals including spin foams, asymptotic safety, ...)
  - Canonical approaches (geometrodynamics, connection dynamics, loop dynamics, ...)
- String theory
- Fundamental discrete approaches
  (quantum topology, causal sets, group field theory, ...);
  have partially grown out of the other approaches

C. Kiefer, *Quantum Gravity* (Oxford 2012)

Gravitons from the early Universe

Gravitons are created out of the vacuum during an inflationary phase of the early Universe ($\sim 10^{-34}$ s after the big bang); the quantized gravitational mode functions $h_{\mathbf{k}}$ in de Sitter space obey

$$\langle h_{\mathbf{k}} h_{\mathbf{k}'} \rangle = \frac{4}{k^3} \, (t_{\mathrm{P}} H)^2 \, \delta(\mathbf{k} + \mathbf{k}') \equiv P_{\mathrm{t}} \; \delta(\mathbf{k} + \mathbf{k}')$$

Power spectrum:

$$\Delta_{\mathrm{t}}^2(k) := \frac{k^3}{2\pi^2} P_{\mathrm{t}} = \frac{2}{\pi^2} \, (t_{\mathrm{P}} H)^2$$

(H is evaluated at Hubble-horizon exit, i.e. at $|k\eta| = 1$)

The BICEP2 experiment

"Background Imaging of Cosmic Extragalactic Polarization"

Figure credit: BICEP2 Collaboration

Most likely, the observed signal comes from a dust foreground (arXiv:1502.00612)

Quantum origin of perturbations

Power spectrum for the scalar modes (inflaton plus metric):

$$\Delta_{\mathrm{s}}^2(k) = \frac{1}{8\pi^2}\,(t_{\mathrm{P}}H)^2\,\epsilon^{-1} \approx 2 \times 10^{-9}$$

ϵ: slow-roll parameter

Tensor-to-scalar ratio: $r := \frac{\Delta_{\mathrm{t}}^2}{\Delta_{\mathrm{s}}^2} = 16\epsilon$

The CMB spectrum from the PLANCK mission

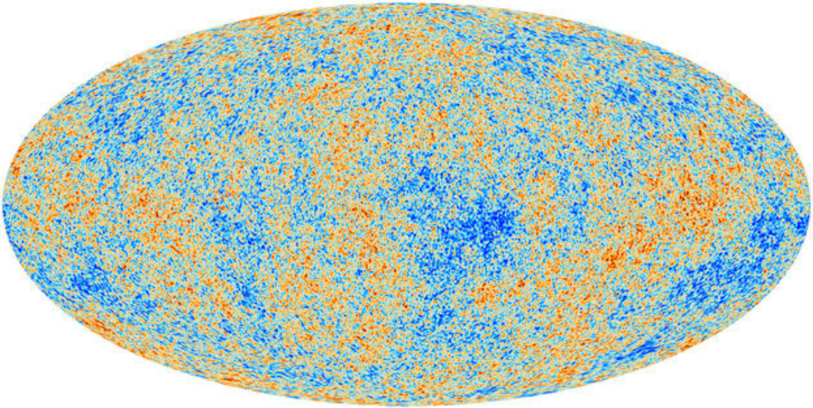
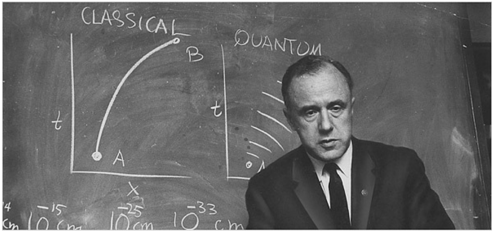


Figure credit: ESA/PLANCK Collaboration

First observational test of quantum gravity

- Within the inflationary scenario, the observed CMB fluctuations can only be understood from quantized metric plus scalar field modes.
- This is an indirect test of linearized quantum gravity.
- The observation of primordial B-modes would be a direct confirmation of the existence of gravitons.
- The difference in the duration of inflation between the 'cold spots' and the 'hot spots' in the CMB spectrum is only of the order of the Planck time.

Quantum geometrodynamics



(a) John Archibald Wheeler

(b) Bryce DeWitt

Application of Schrödinger's procedure to general relativity leads to

$$\hat{H}\Psi \equiv \left(-16\pi G\hbar^2 G_{abcd}\frac{\delta^2}{\delta h_{ab}\delta h_{cd}} - (16\pi G)^{-1}\sqrt{h}\left({}^{(3)}R - 2\Lambda\right)\right)\Psi = 0$$

Wheeler–DeWitt equation

$$\hat{D}^a\Psi \equiv -2\nabla_b\frac{\hbar}{\mathrm{i}}\frac{\delta\Psi}{\delta h_{ab}} = 0$$

quantum diffeomorphism (momentum) constraint

Problem of time

- External time t has vanished from the formalism
- This holds also for loop quantum gravity and probably for string theory
- Wheeler–DeWitt equation has the structure of a wave equation any may therefore allow the introduction of an 'intrinsic time'
- Hilbert-space structure in quantum mechanics is connected with the probability interpretation, in particular with probability conservation *in time* t; what happens with this structure in a timeless situation?
- What is an observable in quantum gravity?

Recovery of quantum field theory in an external spacetime

An expansion of the Wheeler–DeWitt equation with respect to the Planck mass leads to the functional Schrödinger equation for non-gravitational fields in a spacetime that is a solution of Einstein's equations
(Born–Oppenheimer type of approximation)

(*Lapchinsky and Rubakov* 1979, *Banks* 1985, *Halliwell and Hawking* 1985, *Hartle* 1986, *C.K.* 1987, . . .)

Quantum gravitational corrections

Next order in the Born–Oppenheimer approximation gives

$$\hat{H}^{\mathrm{m}} \to \hat{H}^{\mathrm{m}} + \frac{1}{m_{\mathrm{P}}^2}\,(\text{various terms})$$

(*C.K. and Singh* (1991); *Barvinsky and C.K.* (1998))

- Quantum gravitational correction to energy values
- Possible contribution to the CMB anisotropy spectrum (*Brizuela, C.K., Krämer* 2012–2016, ...)

Quantization of a Friedmann Universe

Closed Friedmann–Lemaître universe with scale factor a, containing a homogeneous massive scalar field ϕ (two-dimensional *minisuperspace*)

$$\mathrm{d}s^2 = -N^2(t)\mathrm{d}t^2 + a^2(t)\mathrm{d}\Omega_3^2$$

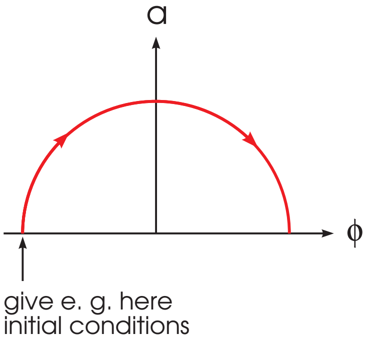
The Wheeler–DeWitt equation reads (with units $2G/3\pi = 1$)

$$\frac{1}{2}\left(\frac{\hbar^2}{a^2}\frac{\partial}{\partial a}\left(a\frac{\partial}{\partial a}\right) - \frac{\hbar^2}{a^3}\frac{\partial^2}{\partial\phi^2} - a + \frac{\Lambda a^3}{3} + m^2a^3\phi^2\right)\psi(a,\phi) = 0$$

Factor ordering chosen in order to achieve covariance in minisuperspace

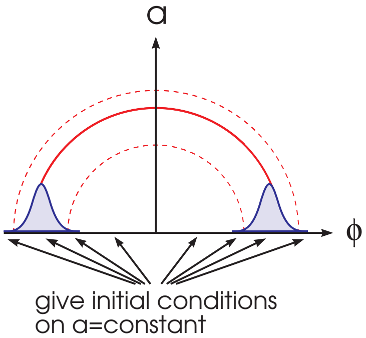
Determinism in classical and quantum theory

Classical theory



Recollapsing part is deterministic successor of expanding part

Quantum theory



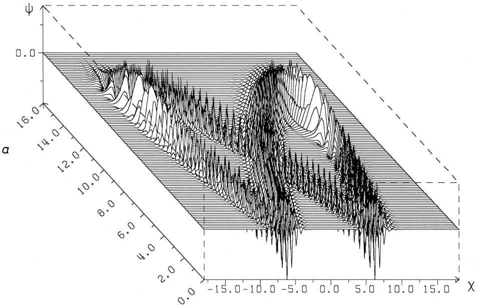
'Recollapsing' wave packet must be present 'initially'

No intrinsic difference between 'big bang' and 'big crunch'!

Example

Indefinite Oscillator

$$\hat{H}\psi(a,\chi) \equiv (-H_a + H_\chi)\psi \equiv \left(\frac{\partial^2}{\partial a^2} - \frac{\partial^2}{\partial \chi^2} - a^2 + \chi^2\right)\psi = 0$$



C. K. (1990)

Singularity avoidance

No general agreement on the criteria!

Sufficient criteria in quantum geometrodynamics:

- Vanishing of the wave function at the point of the classical singularity (dating back to DeWitt 1967)
- Spreading of wave packets when approaching the region of the classical singularity

(These criteria were successfully applied in a number of models by Albarran, Bouhmadi-López, Dąbrowski, Kamenshchik, C.K., Kwidzinski, Krämer, Marto, Moniz, Sandhöfer)

Decoherence in quantum cosmology

In quantum cosmology, arbitrary superpositions of the gravitational field and matter states can occur. How can we understand the emergence of an (approximate) classical Universe?

Introduction of inhomogeneities

Describe small inhomogeneities by multipoles $\{x_n\}$ around the minisuperspace variables (e.g. a and ϕ)

$$\left(H_0 + \sum_n H_n(a, \phi, x_n)\right) \Psi(\alpha, \phi, \{x_n\}) = 0$$

(Halliwell and Hawking 1985)

If ψ_0 is of WKB form, $\psi_0 \approx C \exp(\mathrm{i}S_0/\hbar)$ (with a slowly varying prefactor C), one will get with $\Psi = \psi_0 \prod_n \psi_n$,

$$\mathrm{i}\hbar \frac{\partial \psi_n}{\partial t} \approx H_n \psi_n$$

with

$$\frac{\partial}{\partial t} \equiv \nabla S_0 \cdot \nabla$$

t: 'WKB time' – controls the dynamics in this approximation

Decoherence in quantum cosmology

- 'System': global degrees of freedom (scale factor, inflaton field, ...)
- 'Environment': small density fluctuations, gravitational waves, ...

(Zeh 1986, C.K. 1987)

Example: scale factor a of a de Sitter universe ($a \propto \mathrm{e}^{H_\mathrm{I} t}$) ('system') experiences decoherence by gravitons ('environment') according to

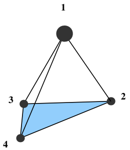
$$\rho_0(a,a') \to \rho_0(a,a') \exp\left(-C H_\mathrm{I}^3 a (a-a')^2\right) \ , \ C > 0$$

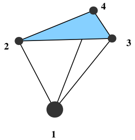
The Universe assumes classical properties at the beginning of inflation

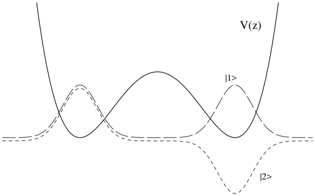
(Barvinsky, Kamenshchik, C.K. 1999)

Time from symmetry breaking

Analogy from molecular physics: emergence of chirality







dynamical origin: decoherence through scattering by light or air molecules

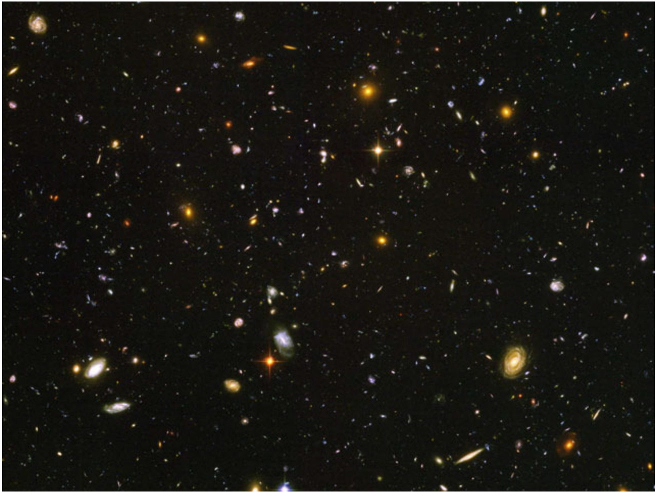
Quantum cosmology: decoherence between $\exp(\mathrm{i}S_0/G\hbar)$- and $\exp(-\mathrm{i}S_0/G\hbar)$-components of the wave function through interaction with e.g. weak gravitational waves

Example for decoherence factor:
$\exp\left(-\frac{\pi m H_0^2 a^3}{128\hbar}\right) \sim \exp\left(-10^{43}\right)$ (C.K. 1992)

Decoherence of primordial fluctuations

During the inflationary phase (ca. 10^{-34} after the Big Bang) there is a quantum-to-classical transition for the ubiquitous fluctuations of the inflaton and the metric.
The process of decoherence is crucial in understanding this transition (C.K., Lohmar, Polarski, Starobinsky 1998, 2007).

The fluctuations then behave like classical stochastic quantities and yield the seeds for the structures in the Universe. Quantum gravity is needed to understand the power spectrum.



How special is the Universe?

Penrose (1981):

Entropy of the observed part of the Universe is maximal if all its mass is in one black hole; the probability for our Universe would then be (updated version from C.K. arXiv:0910.5836)

$$\frac{\exp\left(\frac{S}{k_{\rm B}}\right)}{\exp\left(\frac{S_{\rm max}}{k_{\rm B}}\right)} \sim \frac{\exp\left(3.1\times 10^{104}\right)}{\exp\left(1.8\times 10^{121}\right)} \approx \exp\left(-1.8\times 10^{121}\right)$$

Arrow of time from quantum cosmology

Fundamental asymmetry with respect to "'intrinsic time"':

$$\hat{H}\Psi = \left(\frac{\partial^2}{\partial\alpha^2} + \sum_i \left[-\frac{\partial^2}{\partial x_i^2} + \underbrace{V_i(\alpha, x_i)}_{\to 0 \text{ for } \alpha\to-\infty} \right] \right) \Psi = 0$$

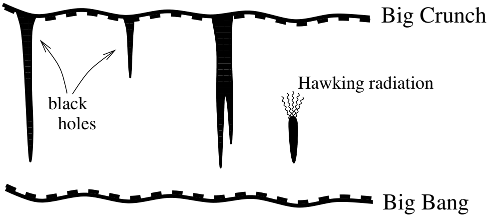
Is compatible with simple boundary condition:

$$\Psi \stackrel{\alpha\to-\infty}{\longrightarrow} \psi_0(\alpha) \prod_i \psi_i(x_i)$$

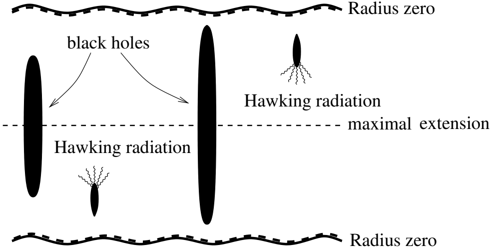
Entropy increases with increasing α, since entanglement with other degrees of freedom increases;
this defines the direction of time

Is the expansion of the Universe a tautology?

Arrow of time in a recollapsing quantum universe



(*Penrose* 1979)



(*C.K. and Zeh* 1995)

Interpretation of quantum cosmology

Almost all approaches to quantum gravity preserve the linear structure of quantum theory and thus the strict validity of the superposition principle.

Main interpretation of quantum cosmology:
Everett interpretation (with decoherence as a key ingredient)

Bryce S. DeWitt 1967:

Everett's view of the world is a very natural one to adopt in the quantum theory of gravity, where one is accustomed to speak without embarassment of the 'wave function of the universe.' It is possible that Everett's view is not only natural but essential.

Conclusion

- At the fundamental level of quantum gravity, there is no need for a probability interpretation, since there exist neither time nor observers.
- Time and observers appear only in the semiclassical limit; classical properties follow through decoherence.
- The probability interpretation is thus needed only in this limit and can perhaps be described in the sense of Zurek (2005).
- The origin of the direction of time can be understood in this framework, at least in principle.