What can black holes teach us about quantum gravity?

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QG

Plan

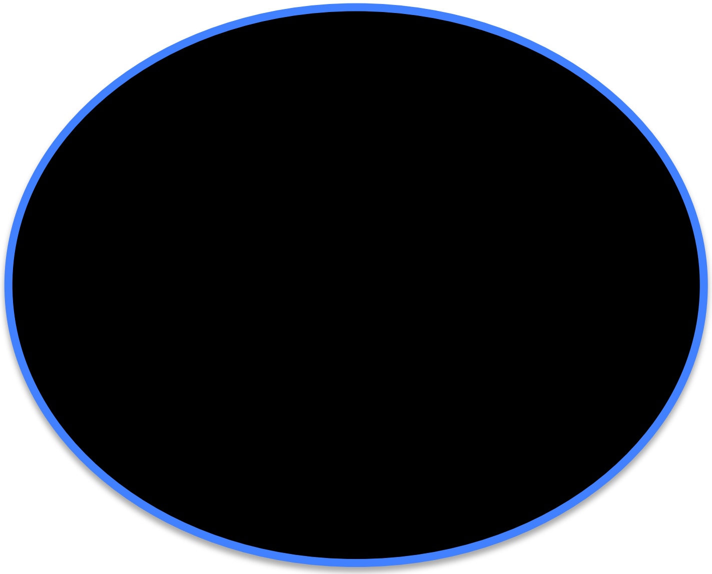
- Black hole entropy

(It's not really about black holes…)

- Black hole information paradox

(It's not really about black holes…)

…but black holes point our noses in fruitful directions…



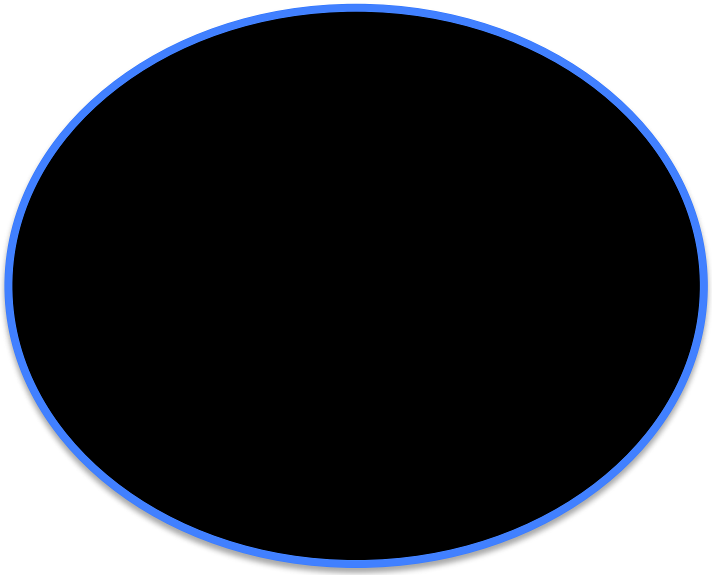
Black hole entropy

Bekenstein, 1972

$$S_{BH} = \alpha(\text{Horizon Area})$$

$$\alpha \sim L_{\text{Planck}}^{-2}$$

$$L_{\text{Planck}}^2 = \hbar G/c^3 = (10^{-33}\,\text{cm})^2$$



Generalized second law:

$$\Delta(S_{\text{outside}} + S_{\text{BH}}) \geq 0$$

Requires that the BH radiates!

Bekenstein's derivation of black hole temperature and entropy

...conceptually intact but streamlined and rephrased a bit...

$$\frac{\kappa}{8\pi G}dA = dM - \Omega_H dJ - \Phi dQ$$

Bekenstein was the first to write this down. He got it from dA(M,J,Q), and didn't know that κ was surface gravity (which is intensive, and constant over the horizon!)

$$\frac{\kappa}{8\pi G}dA = TdS = \delta Q$$

Interpret lhs thermodynamically.

$$T = \delta Q_{\rm min}/\ln 2$$

T is the heat corresponding to one bit of information-theoretic entropy.

Find minimum heat from minimum Killing energy of a neutral particle of mass μ and size b:

$$\delta Q_{\rm min} = (|\chi|\mu)_{\rm min} \sim \kappa\,(b\mu)_{\rm min} \sim \hbar\kappa$$

Hence $T \sim \hbar\kappa, \quad \delta A_{\rm min} \sim \hbar G, \quad \text{and} \quad S \sim A/\hbar G$

He noted that the minimum area change is independent of M,J,Q.

GSL holds only if the black hole radiates at the Hawking temperature,

$$T_H = \frac{\hbar\kappa}{2\pi}, \quad \kappa = \text{surface gravity} = \frac{1}{2R_{\text{horizon}}}$$

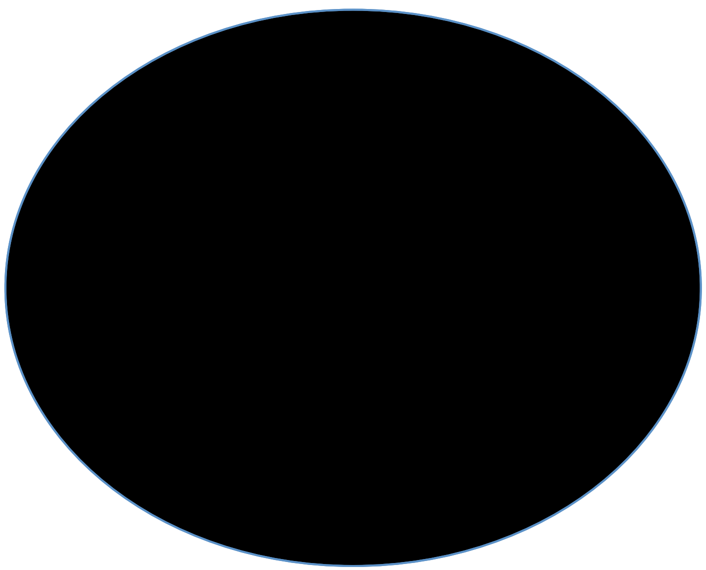
and has entropy $$S_{BH} = \frac{A}{4L_P^2}$$

Hawking radiation:

- is a *quantum field vacuum instability*
- comes from *outside* the black hole
- is correlated to quantum field fluctuations *inside* the black hole

What is black hole entropy counting?

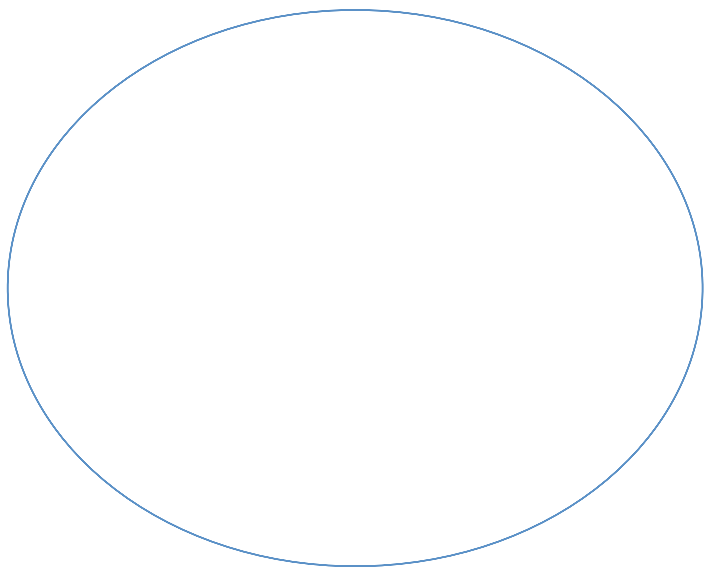
What is the 'source' of black hole thermodynamics?



What is black hole entropy counting?

What is the ‘source’ of black hole thermodynamics?

An horizon is locally just like any other place in spacetime!

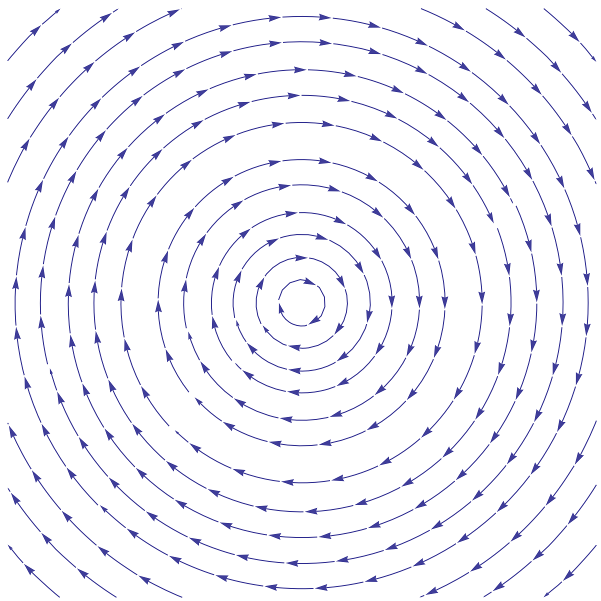


So look for the answers in flat spacetime…

Black hole thermodynamics in flat spacetime

- Bekenstein's derivation of the entropy applies to acceleration horizons.
- The origin of the Hawking effect is the Unruh effect in flat spacetime.
- The Minkowski vacuum has area law entanglement entropy.
- The necessity of variable causal structure and the Einstein equation follows from thermodynamics of the vacuum.
- Newton's constant depends on the matter content of physics, runs with energy, and becomes large at short distances.

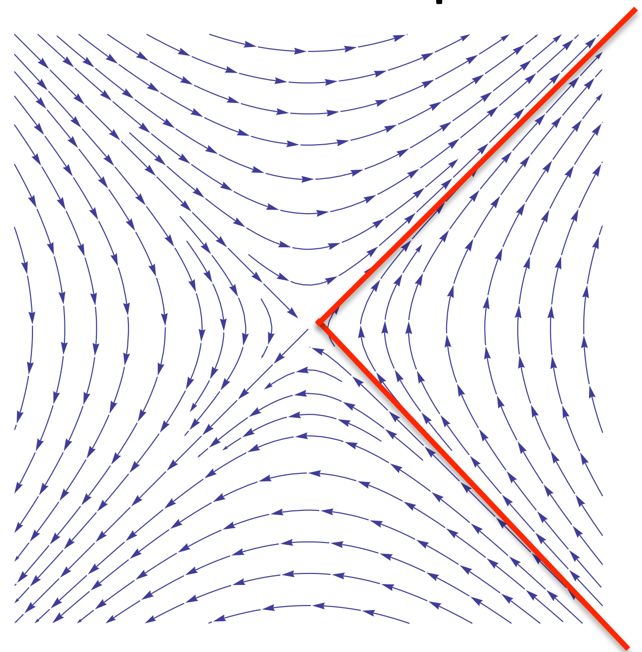
Euclidean space



Rotation symmetry

$$ds^2 = dx^2 + dy^2 = dr^2 + r^2 d\theta^2$$

Minkowski space



Lorentz boost symmetry

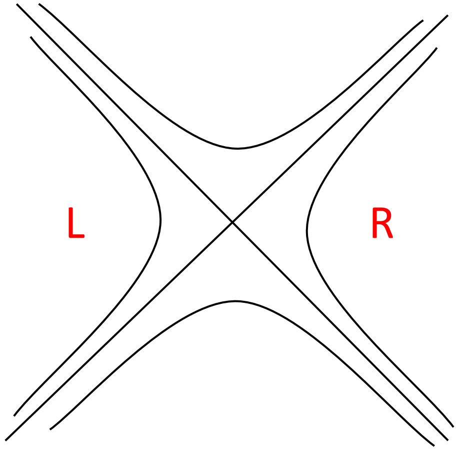
$$ds^2 = dt^2 - dx^2 = l^2 d\eta^2 - dl^2$$

Unruh effect

Lorentz invariance and energy positivity imply the Minkowsi vacuum is a thermal state when restricted to the wedge:

$$\rho_R = Tr_L |0\rangle\langle 0| \propto \exp\left(-\frac{2\pi}{\hbar} H_{Boost}\right)$$

Bisognano-Wichmann (1975), *Davies* (1975), *Unruh* (1976)



A uniformly accelerated observer a distance l from the horizon sees the temperature, $T_{local} = \hbar a / 2\pi = \hbar / 2\pi l.$

Acceleration and T_{local} diverge as l goes to 0.

Vacuum entanglement entropy

(Sorkin '83, Bombelli, Koul, Lee, Sorkin '86)

$$S = -Tr(\rho_R \ln \rho_R)$$

$$\approx \int dA\, dl\, T^3_{\mathrm{local}} \propto \int_{l>\epsilon} dA\, dl\, l^{-3} \propto A/\epsilon^2$$

The hypothesis that black hole entropy is vacuum entanglement entropy has been supported by calculations involving free fields with various regulators, as well as by the Ryu-Takayanagi formula in AdS/CFT. It's not 100% clear however, due to cutoff dependence and issues with how to define entanglement entropy for fields with gauge or diffeomorphism symmetry. Full agreement (when present) requires that spacetime curvature "virial corrections" to the horizon entropy are included.

How is this entanglement entropy *apportioned* in the generalized entropy $S_{\text{outside}} + S_{\text{BH?}}$

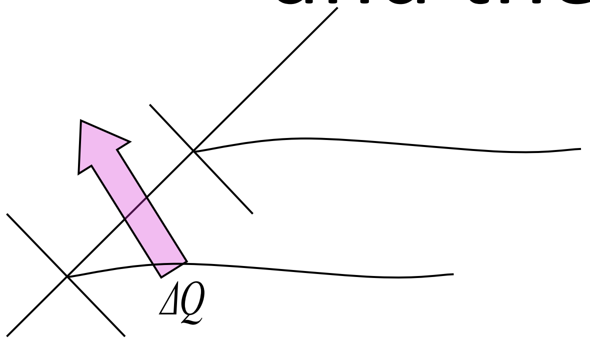
$$S_{\text{tot}} = S_{\text{out}}^{l>\epsilon} + \frac{A}{4\hbar G(\epsilon)} + O(\hbar^0)\,\text{curvature corrections}$$

The total must be independent of ε. As as ε goes to ∞, so G(ε) goes to $G_{N.}$
As ε goes to 0, G(ε) goes to ∞.

But is vacuum entanglement UNIVERSALLY absorbed by G_N?
WHY DOESN'T IT DEPEND ON THE NUMBER OF FIELD SPECIES??

Local causal horizon thermodynamics and the Einstein eqn

(TJ, gr_qc/9504004)



Postulate for __all__ such horizons

1. The horizon system is a 'heat bath', with universal entropy area density. $S = \alpha A$

2. Boost energy flux across the horizon is 'thermalized' at the Unruh temperature. $dS = \delta Q/T$

3. Energy conservation (energy-momentum tensor divergence-free)

⇒ Implies focusing of light rays by spacetime curvature: the causal structure must satisfy Einstein field equation, with Newton's constant $G = \dfrac{1}{4\hbar\alpha}$

Gravity and vacuum entanglement

It seems to follow that:

- Black hole entropy includes -- and may be 100% -- vacuum entanglement
- Infinite entanglement entropy implies G zero.

We have gravity only because the entropy is finite.

Conversely: gravity plausibly cuts off entanglement, when the separation is smaller than the Planck length, by "virtual black holes". (TJ, 1204.6349)

- G depends on the number and species of matter fields. There is no "species problem".
- For a statistical − as opposed to thermodynamic − derivation of the Einstein equation based on a principle of vacuum "entanglement equilibrum" in small geodesic balls at fixed spatial volume, see (TJ 1505.04753).
- N.B. Although the Einstein equation follows from thermodynamics of a vacuum subsystem, QGR is almost certainly an *effective* quantum field theory… but it seems dissipative effects should be expected at the Planck scale.

What about BH microstate counting?

String theory: Doesn't count black hole states. Counts string states on D-branes at weak string coupling. Uses SUSY to link that to the number of states at strong coupling, with the same charges, at which there is a black hole. Newton's constant is not renormalized, thanks to SUSY, so the result of this UV microstate counting can match the Bekenstein-Hawking entropy perfectly.

Loop quantum gravity: Counts black hole states, identified as spin networks. Result arises from a kind of entanglement entropy, and scales with area, but depends on a free (Immirzi) parameter. Newton's constant is (presumably) renormalized, so can't directly compare coefficient with BH entropy. Also the fact that the result Is the same for BH coupled to Maxwell field seems (to me) problematic, in view of the different running of G. (TJ, 0707.4026)