Spacetime in String Theory

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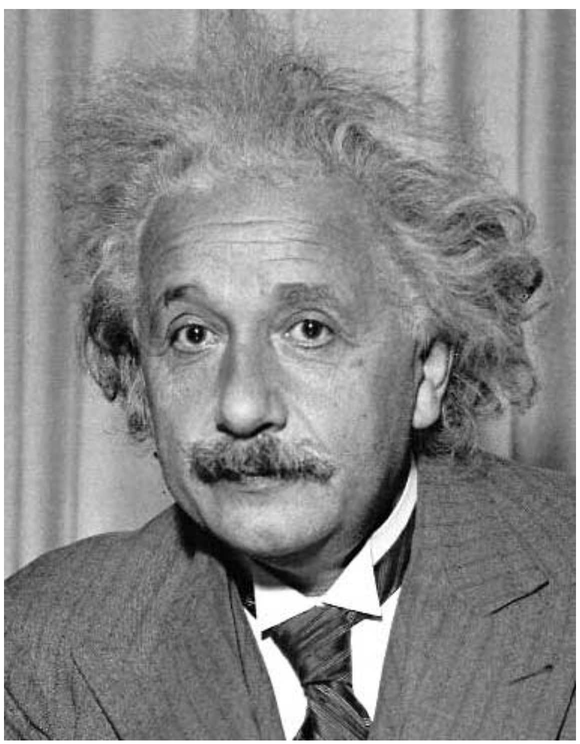
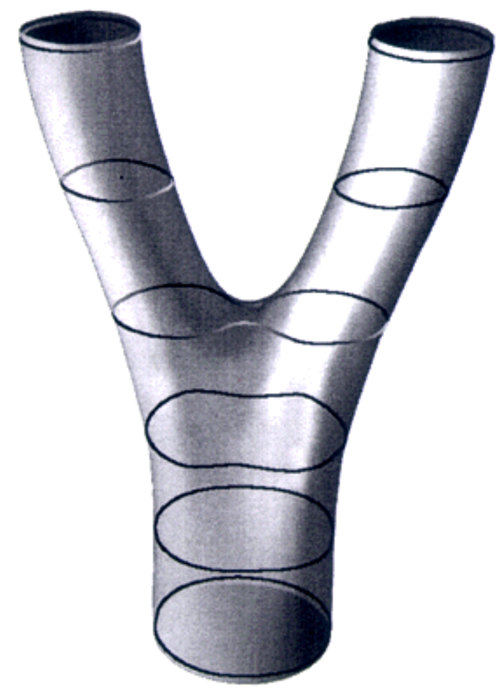




String theory starts with:

Point particles

Strings

This leads to :

1) Higher dimensions

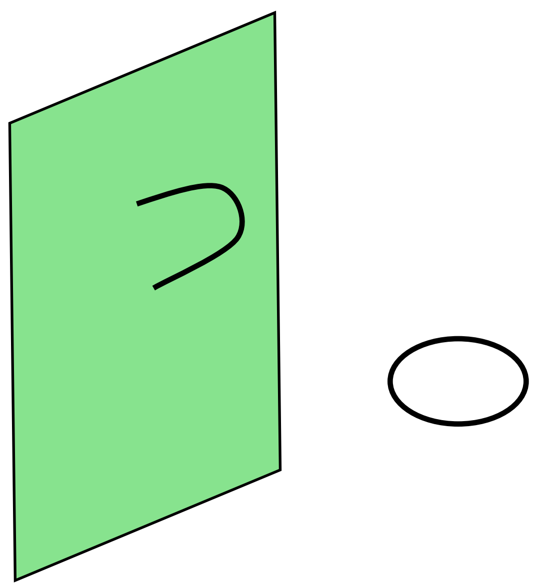
2) Supersymmetry

Supersymmetry is not just a symmetry between bosons and fermions. It is an extension of the Poincare symmetry of spacetime.

It represents the first extension of spacetime since space and time were unified by Minkowski.

String theory is not just a theory of strings.

There are other extended objects - branes



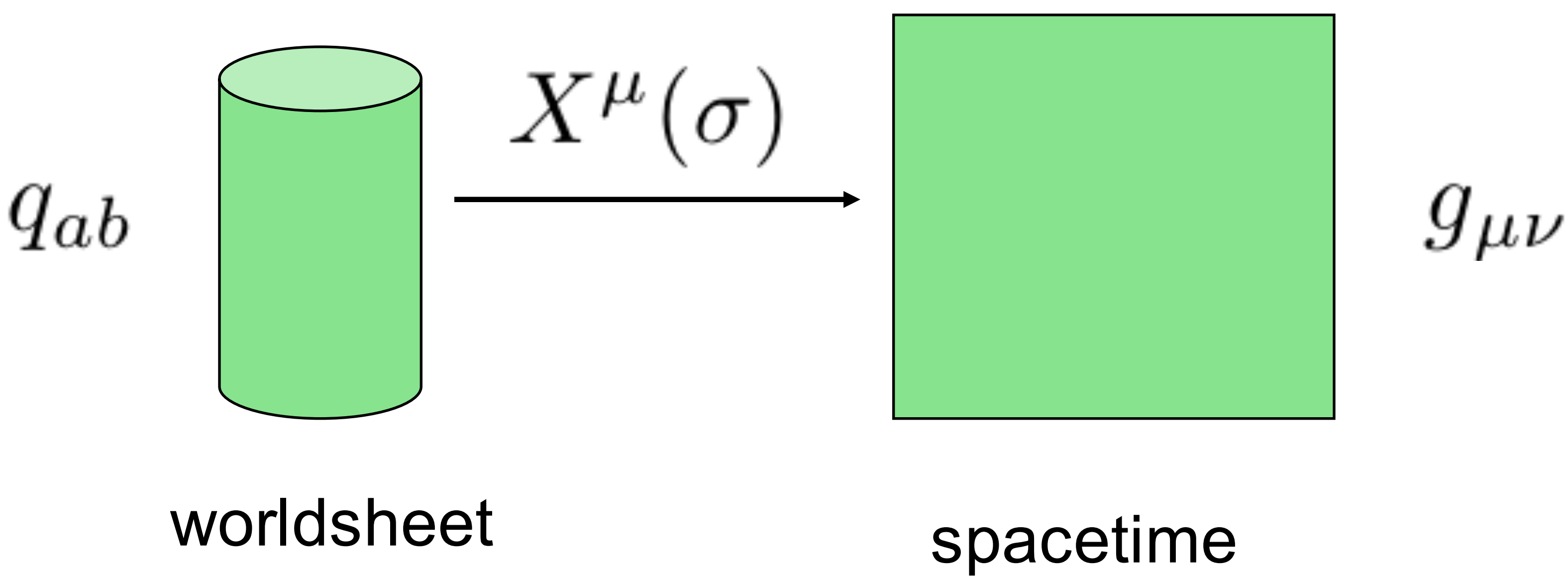
Branes are surfaces on which open strings can end. We might be living on a three-brane in a higher dimensional space.

Spacetime in String Theory

- Perturbative theory
  - a) Geometrically different spacetimes can be equivalent
  - b) Some singularities can be resolved
  - c) Topology of space can change
- Nonperturbative theory - holography
  - a) Gauge/gravity duality
  - b) Implications for topology & singularities
  - c) Emergence of spacetime geometry

Perturbative string theory

A string traces out a two dimensional worldsheet, so its dynamics is described by a two dimensional theory.



This is described by a sigma model:

$$S = L_s^{-2} \int d^2\sigma \sqrt{-q}\, q^{ab} \nabla_a X^\mu \nabla_b X^\nu g_{\mu\nu}$$

This action is classically invariant under rescaling q_{ab}. Requiring that this symmetry remain quantum mechanically yields a field equation on the spacetime metric, which is Einstein's equation to lowest order.

Different topologies for the string worldsheet correspond to different orders in a quantum loop expansion.

The role of spacetime has changed dramatically. The spacetime metric acts like “coupling constants” in this 2D theory.

Since the string only senses spacetime through this sigma model, two metrics which yield the same sigma model are indistinguishable in string theory.

Apparently trivial changes to the sigma model can result in dramatic changes to spacetime.

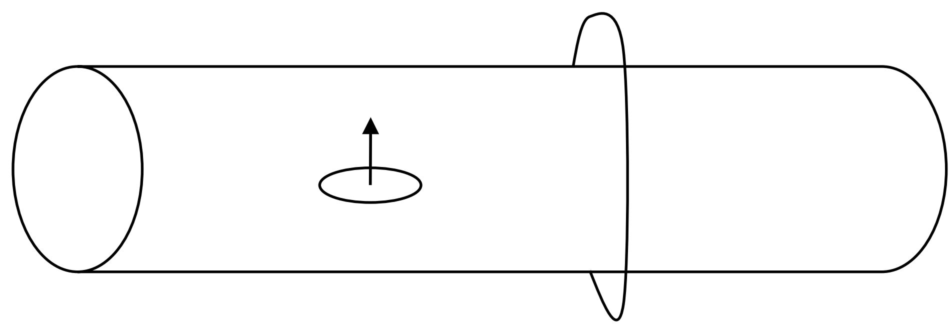
Geometrically different spacetimes can be equivalent:

T - duality

If the spacetime has a symmetry around a circle, a “change of variables” in the sigma model produces a new spacetime where the circle has a different radius.

E.g. flat spacetime compactified on a circle of radius R is equivalent to one with radius L_s^2 / R

Physically this is a result of the fact that strings have winding modes as well as momentum modes



Topologically different spacetimes can be equivalent:

Mirror symmetry

Changing a sign in the sigma model changes its interpretation from strings on a spacetime M_4 x K to strings on M_4 x K' where K and K' are topologically different (Calabi-Yau) spaces.

Dramatic prediction: The number of ways to map S^2 into a Calabi-Yau space using eqs. of degree n (Candelas et al.)

n=1	2875	n=3	317,206,375
n=2	609,250	n=4	242,467,530,000

String theory can resolve certain singularities

- Conical singularities (orbifolds)
- Timelike singularities associated with extended objects (branes)
- Singularities associated with certain types of topology change

The mass of a string wound around a circle of radius R has two contributions:

$$M^2 = \frac{R^2}{L_s^4} - \frac{1}{L_s^2}$$

usual tension

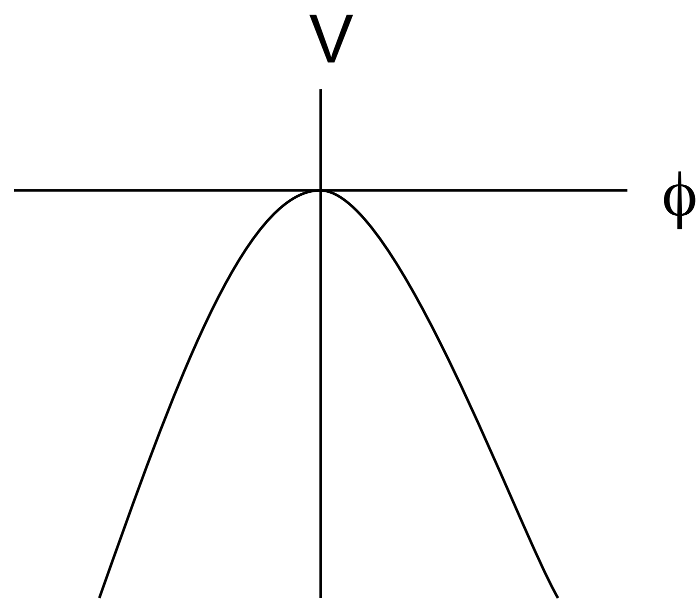
zero point energy

So if $R < L_s$, these wound strings become tachyonic. Tachyons should not be thought of as particles traveling faster than light.

Tachyons just indicate an instability.

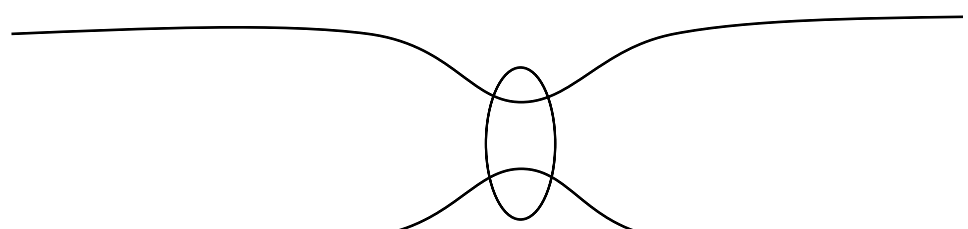
In ordinary field theory, if

$$V(\phi) = -m^2\phi^2$$



$\phi = 0$ is unstable and ϕ rolls down the potential. One says that "tachyons condense".

The topology of space can change in string theory (Silverstein et al.)



Strings wound around the neck become tachyonic when the size is of order L_s.

The result of this instability is that the neck pinches off.

String theory does not resolve all singularities

(Nonlinear) gravitational plane waves are exact solutions to string theory with arbitrary amplitude. If this amplitude diverges, the solution is singular even in string theory.

If all singularities were resolved, spacetimes with negative total energy would be allowed. Minkowski spacetime would be unstable!

We will discuss cosmological and black hole singularities later.

Nonperturbative Theory

Main lesson is holography:

Physics in a region is completely described by fundamental degrees of freedom living on the boundary.

Gauge/gravity duality

(Maldacena; Gubser, Klebanov, Polyakov; Witten)

With anti-de Sitter boundary conditions, string theory (which includes gravity) is completely equivalent to a (nongravitational) gauge theory living on the boundary at infinity.

When string theory is weakly coupled, gauge theory is strongly coupled, and vice versa.

In coordinates that cover the entire spacetime, AdS takes the form:

$$ds^2 = -(r^2+1)dt^2 + \frac{dr^2}{(r^2+1)} + r^2 d\Omega^2$$

The boundary at infinity is a static cylinder

$$ds^2 = -dt^2 + d\Omega^2$$

A light ray can reach infinity and return in finite time.

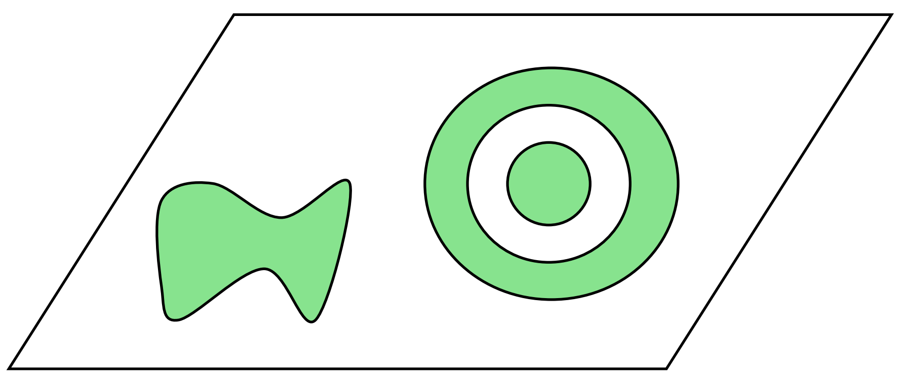
Soup is physical spacetime

Label is gauge theory

One piece of evidence

Explicit correspondence has been found for all states preserving half the SUSY (Maldacena et al.)

Gauge theory: All states created by a single homogeneous field - N x N matrix. This matrix model can be quantized exactly and states are labeled by closed curves in a plane.



String theory: For each state there is a corresponding gravity solution. It is stationary and nonsingular, but can have complicated topology. It is characterized by a solution to a 3D linear equation. The boundary condition for the linear equation is again closed curves in a plane.

The curvature is everywhere below the Planck scale if the area enclosed by each loop is large enough.

The full quantum description of this sector of the theory is given by the matrix model.

Spacetime topology can be ambiguous in string theory

(Berenstein and Miller, 2017)

There are two topologically different classical spacetimes with the following surprising property:

If you quantize the fluctuations about each of them, the resulting Hilbert spaces have states in common. So

There are states which can be given two different spacetime topology interpretations.

A powerful feature of gauge/gravity duality is that statements that are easy to establish on one side often imply highly nontrivial results about the dual theory.

For example, the fact that black hole evaporation must be unitary follows immediately from unitary evolution of the dual gauge theory.

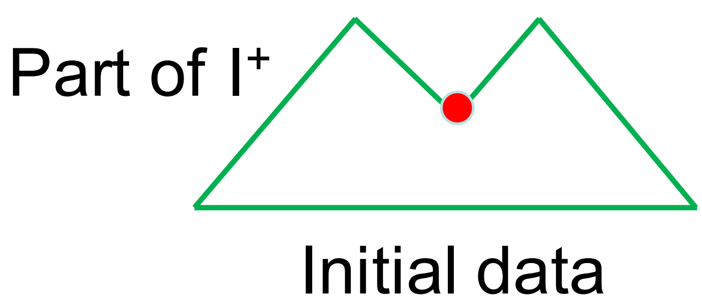
Some Implications of Holography for Spacetime Singularities

(Engelhardt, G.H., 2015)

Cosmic censorship

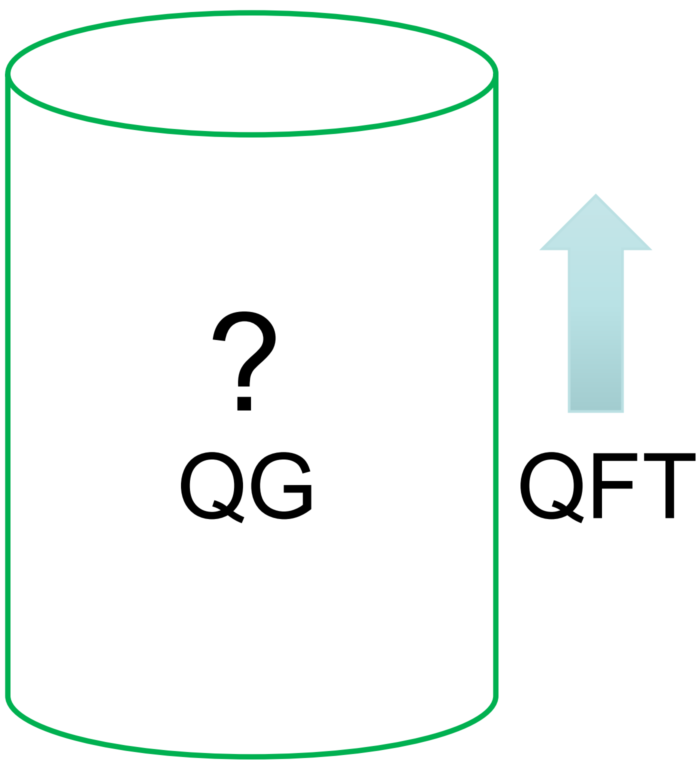
Classical GR conjecture: Generic, asymptotically flat, initial data has a maximal evolution that contains a complete null infinity.

i.e. this can't happen:



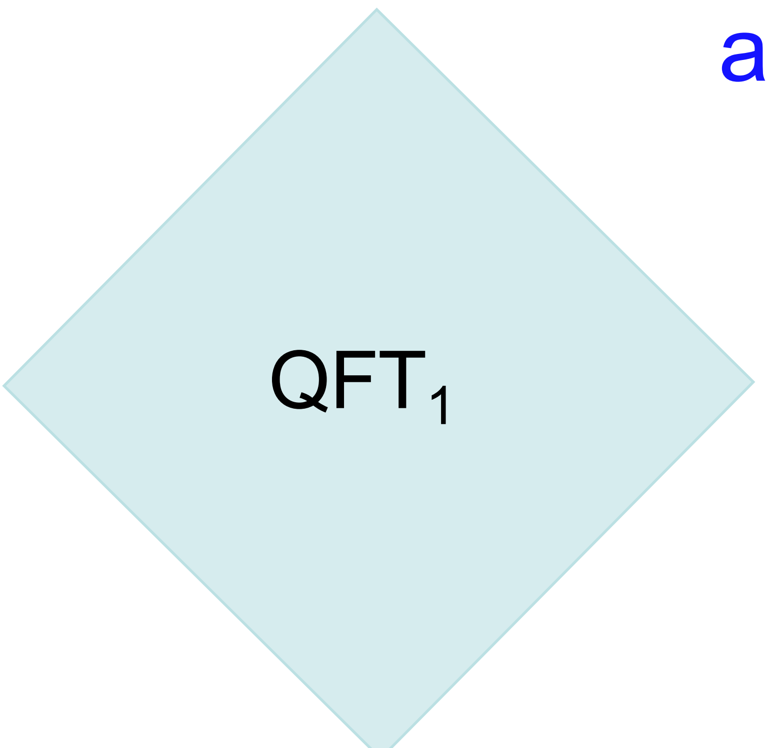
If cosmic censorship fails, it was hoped that quantum gravity would resolve the singularity so evolution continues.

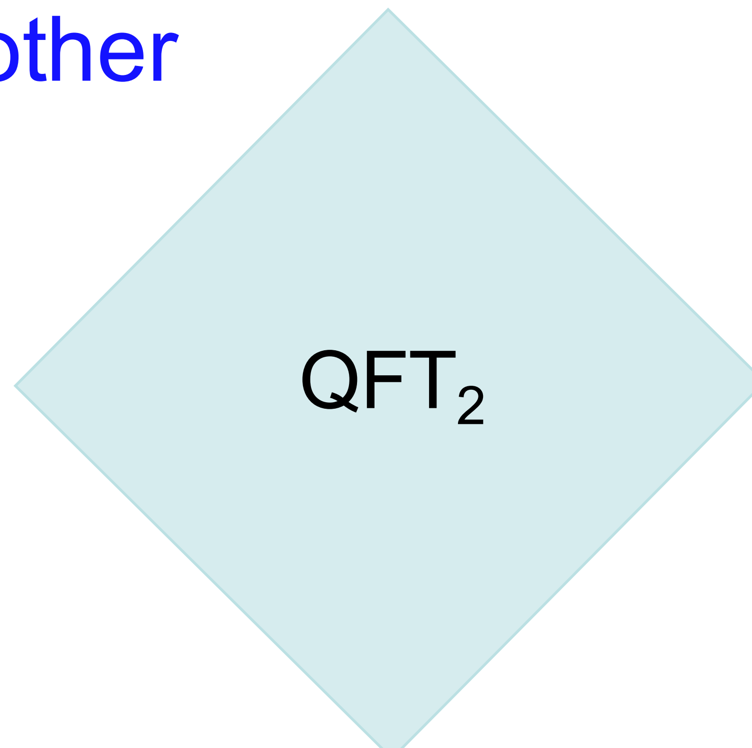
In holography we know that this is true! Regardless of what happens in a localized region in the interior, evolution in the QFT on the boundary continues.



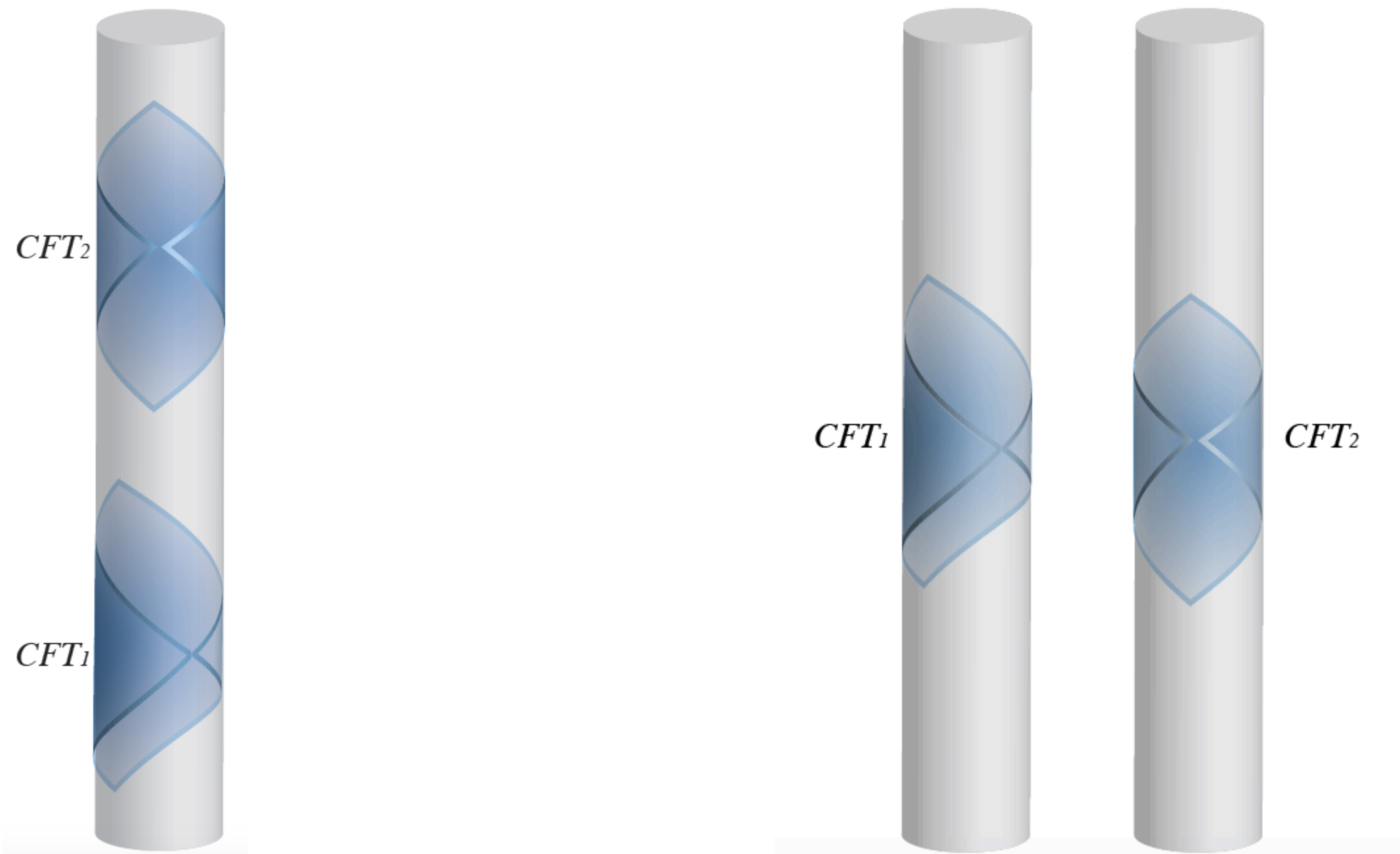
When can two QFTs communicate?

Usually, two QFTs on separate spacetimes cannot send signals to one another





Two copies of a CFT on Minkowski space can be mapped either to one static cylinder or two.

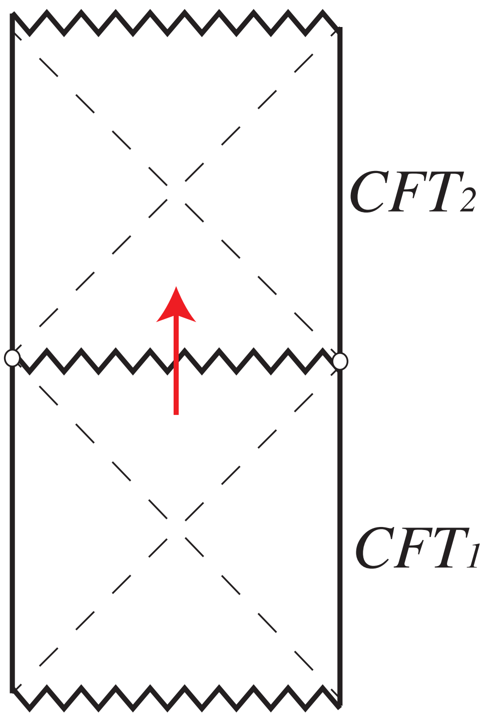


We will consider CFTs on S^2 x R where this problem doesn't arise:

Two CFT's on S^2 x R cannot be conformally mapped into a single larger spacetime, since S^2 x R is conformally maximally extended.

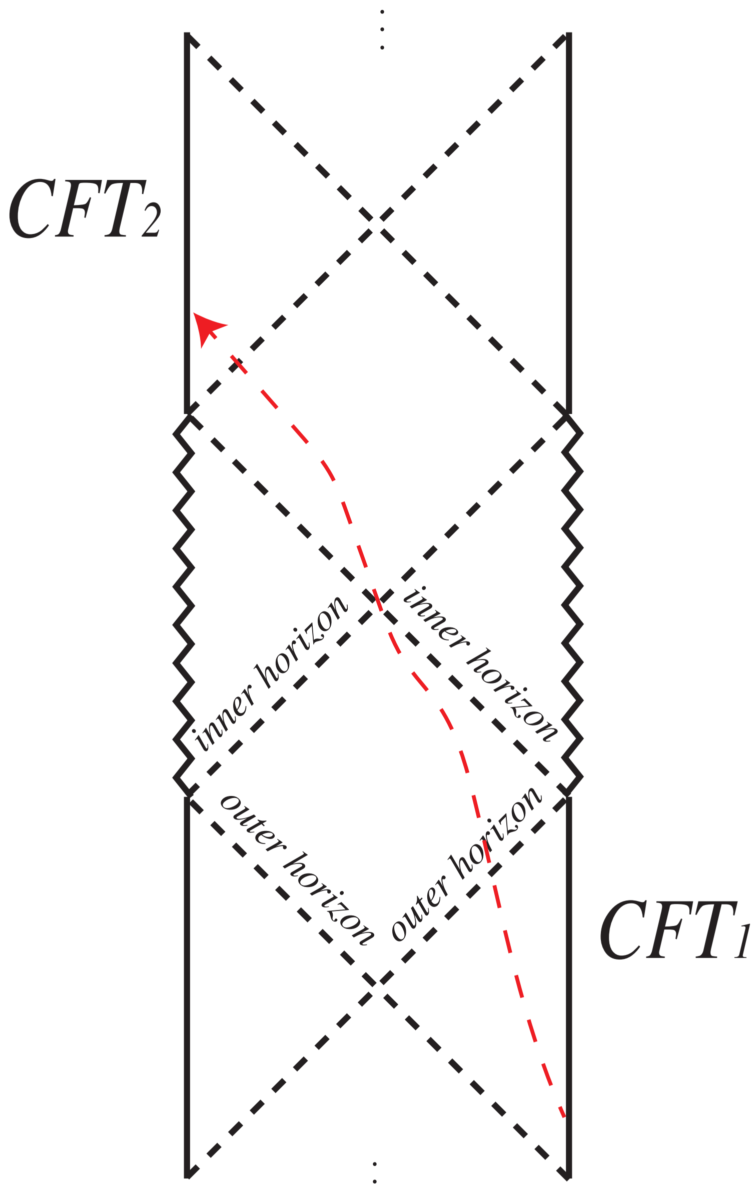
No Transmission Principle (NTP):
If two CFTs on S^2 x R have gravity duals, then no signals can be transmitted between their bulk duals.

No evolution through black holes



Could quantum gravity resolve the singularity and allow signals to emerge in another asymptotically AdS spacetime?

No. This would violate the NTP.



A charged (or rotating) AdS black hole seems to violate the NTP even classically.

But the inner horizon is known to be unstable. Signals cannot get through classically.

NTP implies that signals cannot get through even in full quantum gravity.

Application to singular CFTs

Some CFTs cannot be evolved past a certain time.

If evolution on the boundary stops, then evolution in the bulk must stop as well.

There must be a cosmological singularity classically, which quantum gravity cannot resolve into a bounce.

Examples of singular CFTs

1) Put any CFT on a spacetime with a cosmological singularity (Das, Michelson, Narayan, Trivedi; Engelhardt, Hertog, G.H.)

2) Destabilize a CFT by adding a potential unbounded from below (Hertog, G.H.; Craps, Hertog, Turok; Barbon, Rabinovici)

3) Add a relevant perturbation to a CFT on de Sitter (Maldacena; Harlow, Susskind; Barbon, Rabinovici)

Comments

1) One cannot avoid our conclusions by adding couplings between the CFTs associated with different asymptotic regions, since that would violate causality.

2) One could make up a rule to identify a state in CFT_1 with one in CFT_2, but it would be extra input not contained in the original CFTs – bulk and boundary theories would not be equivalent.

3) There is no natural way to identify the states.

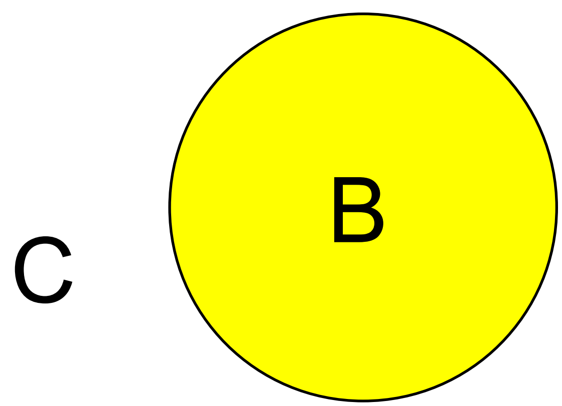
How does Spacetime Emerge From the Dual Gauge Theory?

One expects to recover a familiar semiclassical spacetime in the bulk in a large N limit where the gauge group in the dual theory is SU(N).

The role of entanglement

Given a state $| \Psi >$ and region of space B with complement C, the information inside B is contained in the density matrix

$$\rho_B = Tr_C \, | \Psi > < \Psi |$$



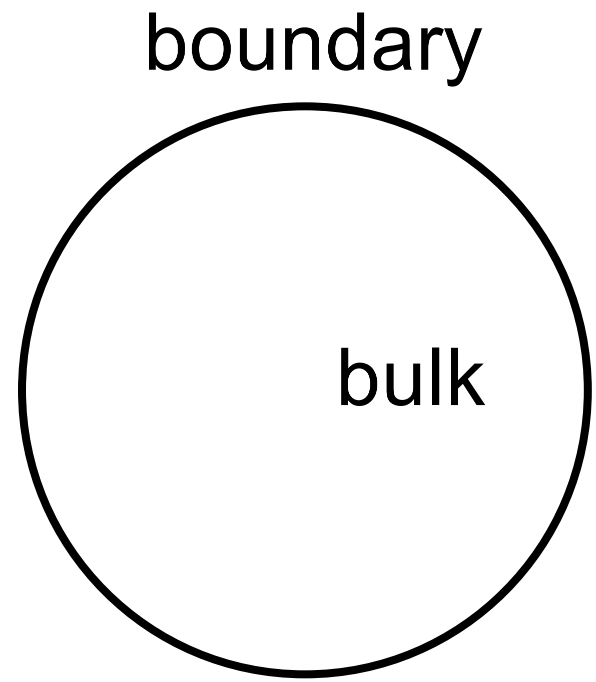
The entanglement entropy is

$$S_{EE} = - Tr \, (\rho_B \log \rho_B)$$

In holography, this can be computed from the area of the extremal bulk surface which ends on the boundary of B (Ryu, Takayanagi; Hubeny, Rangamani, Takayanagi):

$$S_{EE} = A / 4$$

Example: In the CFT ground state, the bulk is AdS.



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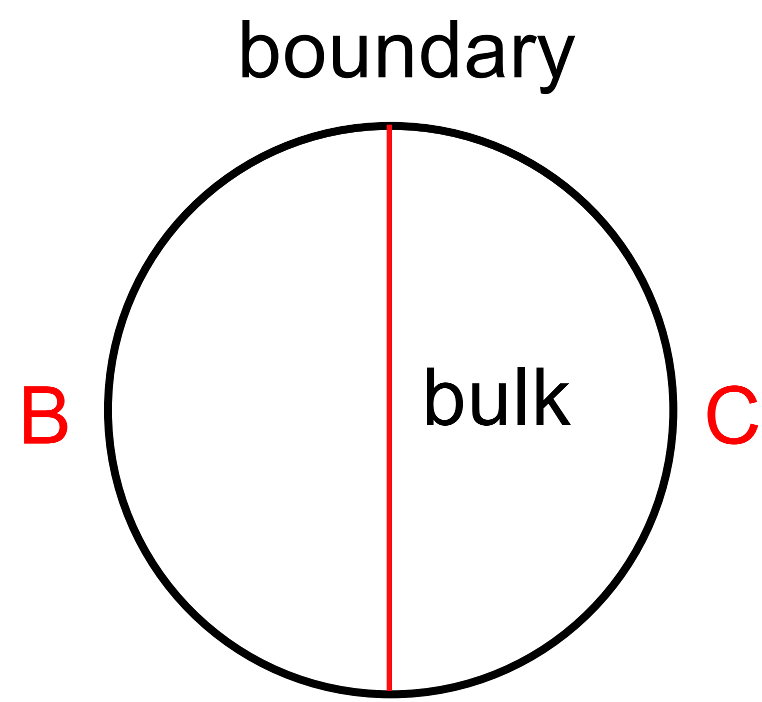
Example: In the CFT ground state, the bulk is AdS. If B is a hemisphere, extremal surface is in red.



Entanglement is important to reconstruct spacetime
(van Raamsdonk, …)

1) If you change the state to decrease entanglement between B and C, the area will decrease – space will start to pinch off.
2) A state with no entanglement between B and C will have diverging T_{ab} at the boundary. This implies singularities in the bulk metric.
3) Can reconstruct the Einstein equation from properties of S_{EE} (to first and second order).

What is spacetime in string theory?

- Perturbatively, the metric acts like coupling constants in a 2D field theory.
- Nonperturbatively, spacetime emerges from a holographic dual theory (at least with AdS boundary conditions).
- Topology can change and may not even be well defined.
- One cannot pass through certain cosmological or black hole singularities.

The picture of spacetime in string theory is still incomplete, but looks very different from general relativity.

But new pieces are filled in every day…