Holographic Dualities and **Quantum Gravity**

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Quantum Gravity: Physics and Philosophy

ERC Project Philosophy of Canonical Quantum Gravity

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Contents

see also talks of S. De Haro
T. Jacobson and G. Horowitz

1. A little history + basic idea

The starting point: a magic formula for the entropy

of (large) **Black Holes**

$$S_{BH} = \frac{c^3}{G\hbar}\ \frac{1}{4}(Area_H)$$

Bekenstein '72

Hawking '74

– BHs look to outside observers like thermodynamical systems

– Matter that fell in, and is veiled to the outside, carries entropy

– The entropy has a **universal geometric** form

for all gravitational theories,
& in any dimension !

– The entropy has a **quantum origin**

BH formula makes no sense for $\hbar = 0$

ENTROPY is an intuitively elusive concept of physics

a measure of information

Boltzmann

Shannon

For a binary digit (q-bit, spin 1/2, coin flip)

$$S = -p_0 \log p_0 - p_1 \log p_1$$

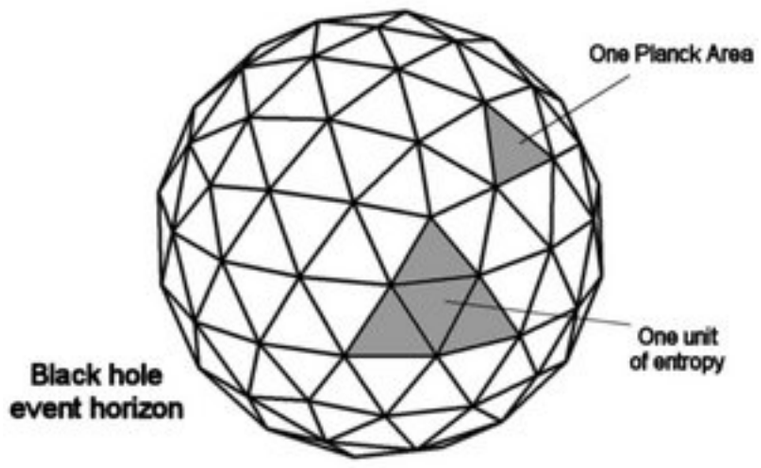
$$S = \begin{cases} 0 & \textbf{frozen} \text{ (certainty)} \quad T = 0 \\ \log 2 & \textbf{maximal capacity} \text{ (uncertainty)} \quad T = \infty \end{cases}$$

NB: The normalization of S is a convention, chemists multiply it by k_B

Key property of entropy: it is extensive

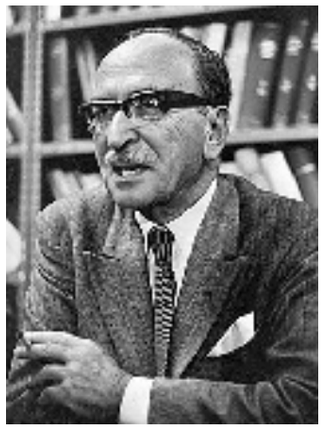
$\underbrace{110}_{N}$ q-bits have information capacity $N \log 2$

« Normally » $N \propto Volume$



but BH formula as if all information was **stored on the Horizon**

BH interior NOT like normal space; entropy bounds



Gabor
’48 ’49

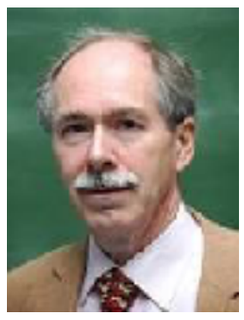
Holographic screen:

(by recording phases as well as amplitudes)
the screen stores
3d information on a 2d surface

Idea:

BH Horizon is a Holographic screen:

‘It should be possible to describe physics with a (2+1)d theory’



’t Hooft ‘93

Susskind ‘94

Ideas can be nice, but their importance is hard to gauge
before they are formulated as a precise mathematical statement

For holographic duality this hapenned with
the famous paper of **Juan Maldacena**

« **The Large N limit of superconformal field theories and supergravity** »

arXiv hep-th/9711200

+ two companion papers:

Gubser, Klebanov, Polyakov « **Gauge theory correlators from non-critical string theory** »

arXiv:hep-th/9802109

Witten « **Anti-de Sitter space and holography** » arXiv:hep-th/9802150

Several important earlier insights `fell in place:'

Membrane paradigm	(fluid/gravity)	**Damour '78** **Price & Thorne '86**
Asymptotic symmetries	(AdS/CFT)	**Brown & Henneaux '86**
1/N expansion	(Yang-Mills/string)	**'t Hooft '74**
Liouville mode as holograhic coordinate		**Polyakov < '97**

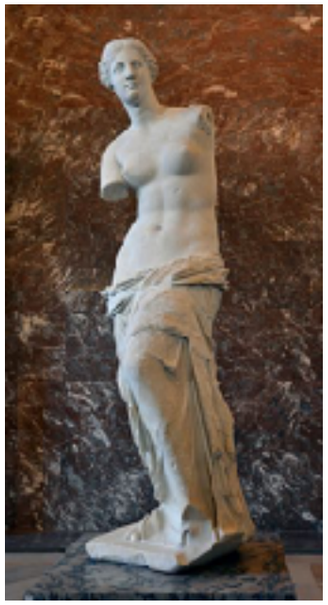
2. AdS/CFT correspondence

following Ariadne's thread

Let me try to describe now the precise statement of **holographic duality**, and the road that led to the formulation of this conjecture

Duality:

2 or more mathematical descriptions of the same physical object



Famous precedent: **Particle-Wave** duality in QM

An **electron** is neither a `particle' nor a `wave', but one or the other description can be a good approximation in a given experiment

$$\Delta x \, \Delta v \sim \frac{\hbar}{mass}$$

For a molecule of 10000 atomic units: $\Delta x \, \Delta v \sim 4 \times 10^{-11} \, m^2/s$

Yet in generalized double-slit experiments such molecules can be shown to behave like waves

Quantum interference of large organic molecules

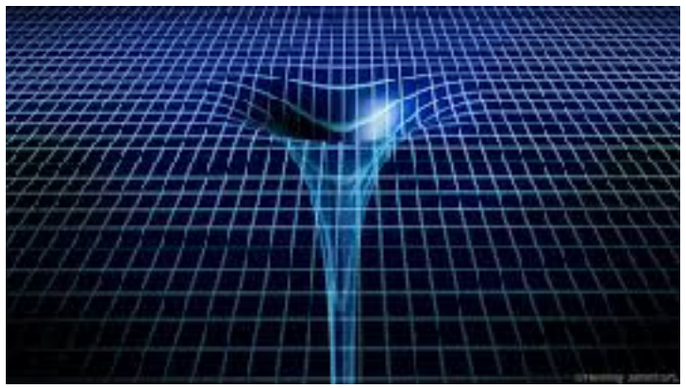
Gerlich, Eibenberger, Tomandl, Nimmrichter, Hornberger, Fagan, Tüxen, Mayor & Arndt
Nature Communications **2**, Article number: 263 (2011)

We will return to this analogy in the end

In the case at hand, the physical object is a **Black Hole**

(or rather, an idealized version of BH)

which one views from two different perspectives



The paradigm was developped in a very dense (and very exciting) five-year period ('93-'98) and it is impossible to do justice to all the important contributions. Here are some key points:

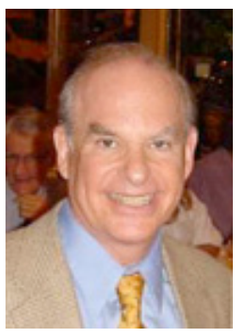
By the early 90's there was substantial accumulated evidence that

String theory is a **perturbatively** consistent theory of QG

from which **variants of** Einstein's theory arise as low-E limits

Yoneya

Scherk



Schwarz

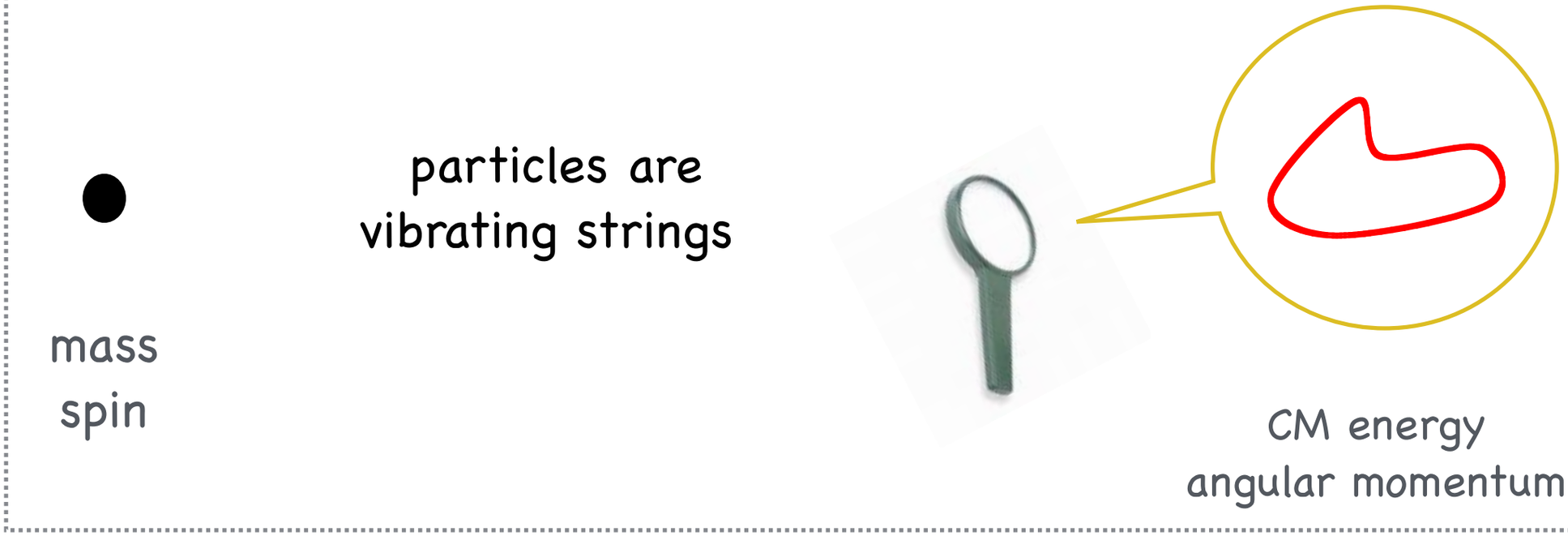
Green

Y; SS '74

GS '84

`perturbative' : small fluctuations around a given classical background, (Minkowski, AdS)x compact

cf Gabriele's talk



The known backgrounds are NOT our world

de Sitter; no 5th force

- there is no obstruction of principle

But:

- differences may not be important for **certain** conceptual puzzles of QG

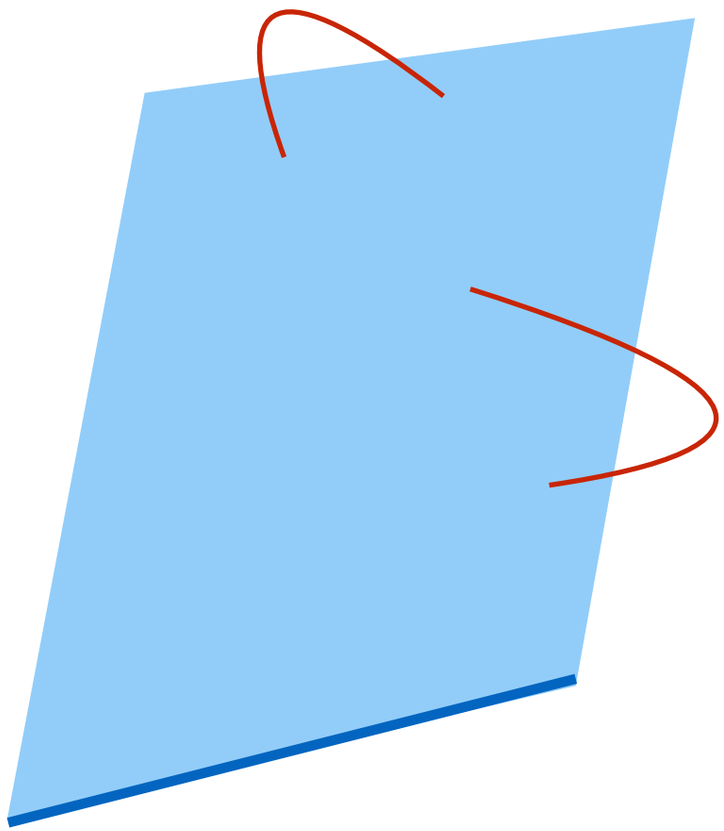
Non-linear theories often admit **soliton** excitations

i.e. stable localized lumps of energy, solutions of the non-linear field equations (magnetic poles, cosmic strings, tsunamis ?)

In string theory these have a striking description as **D-branes**

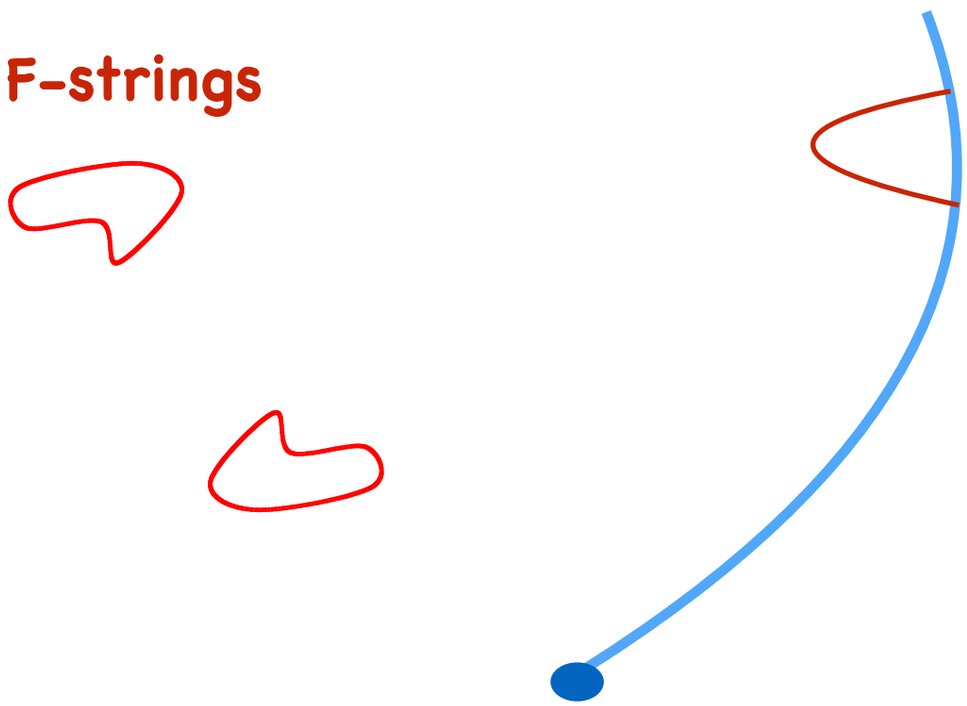
`closed strings can break open on a D-brane'

Polchinski '95



D-string

F-strings

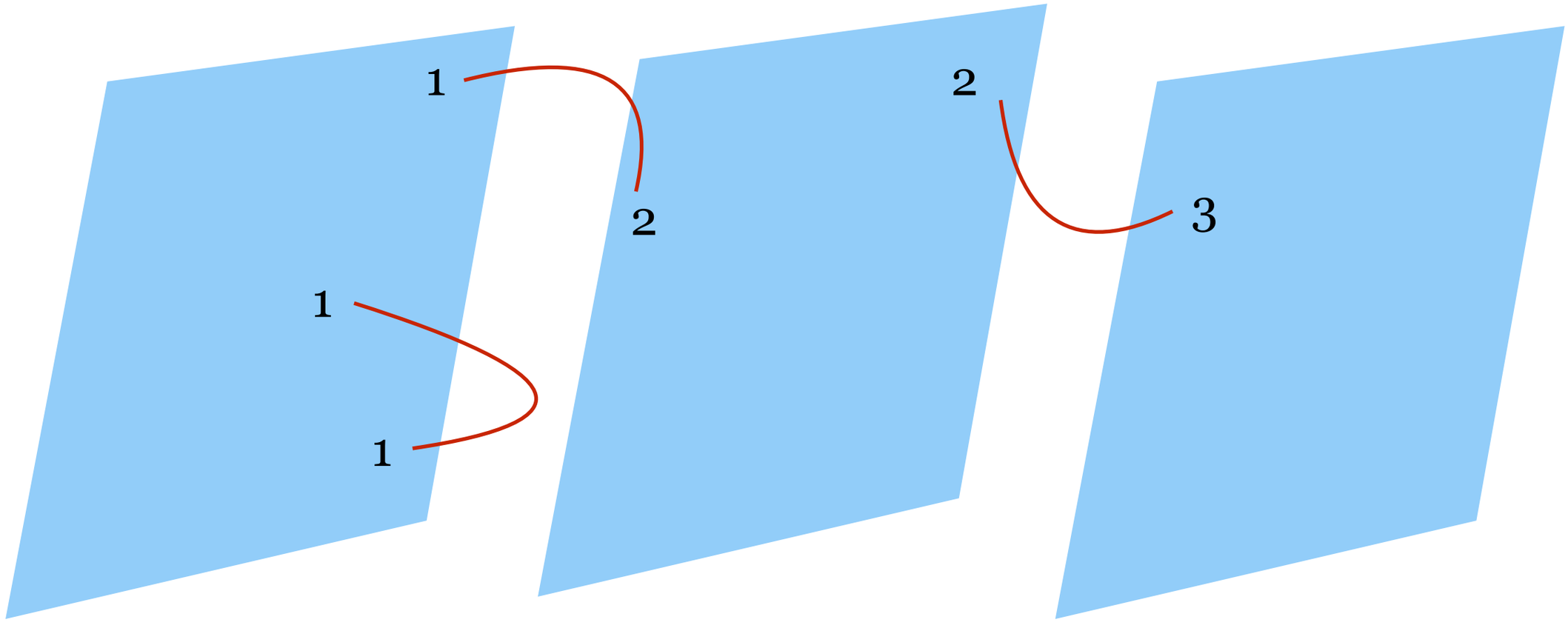


D-particle

time

Don't get fooled: drawing defines unambiguously all properties of these solitons (mass, charges, dynamics)

Not a model, no adjustable parameters; `mathematical inference'



Key observation: Open strings are matrices

The low-E limit of open string theory is a spin-1 gauge theory à la **Yang + Mills**

the cornerstone of the Standard Model

Neveu Scherk '72

We have seen that closed-string theory has solitons on which lives a Yang-Mills theory

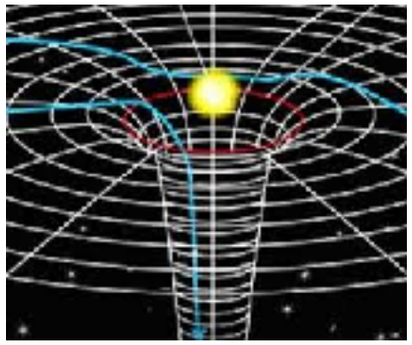
At low E closed-string theory reduces to a (variant of) Einstein's theory of gravity

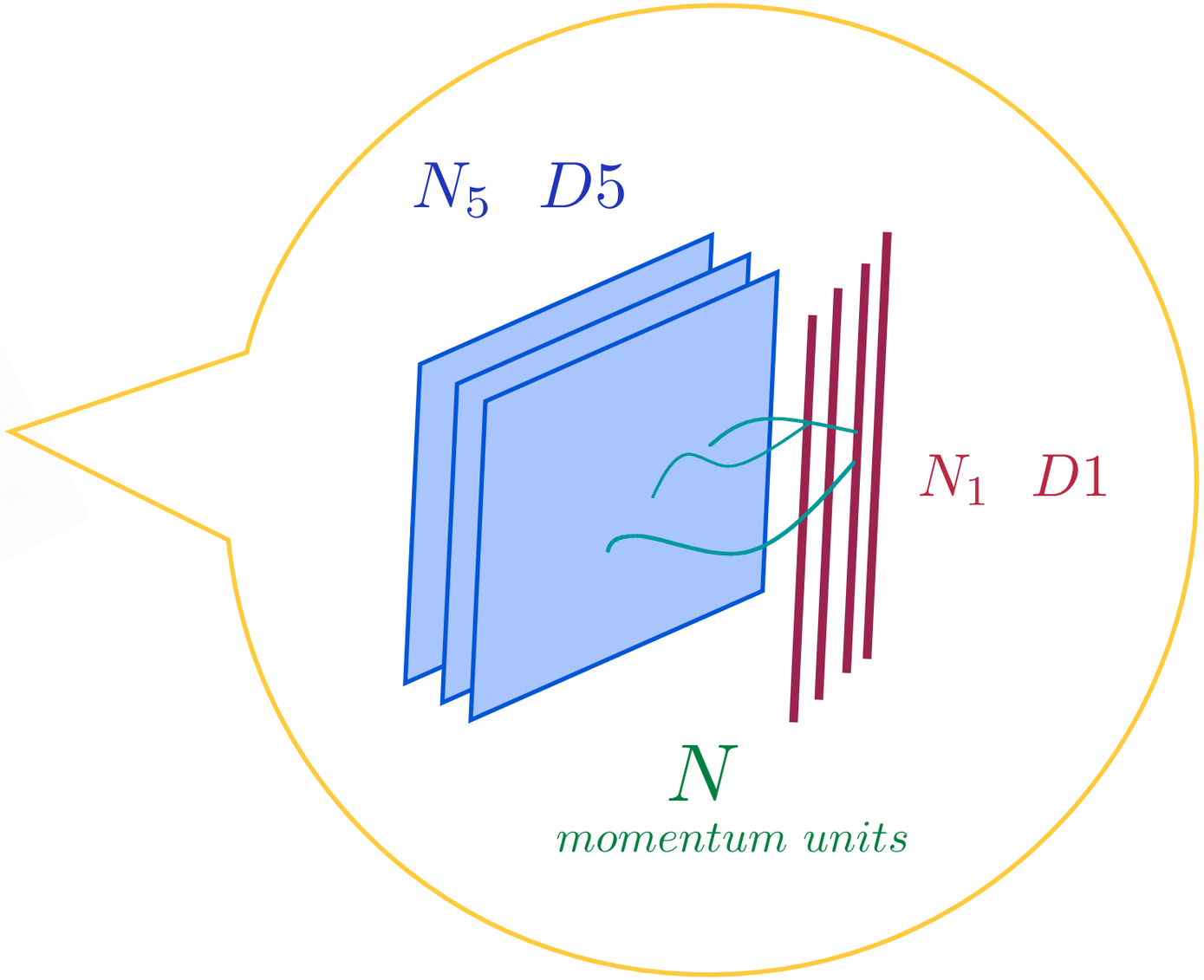
But (generic) gravitational solitons are Black Holes *

∴ Black Holes can be `described' by YM theory

* At extremality and in higher dimensions there exist smooth horizonless `fuzzball' solutions

BH

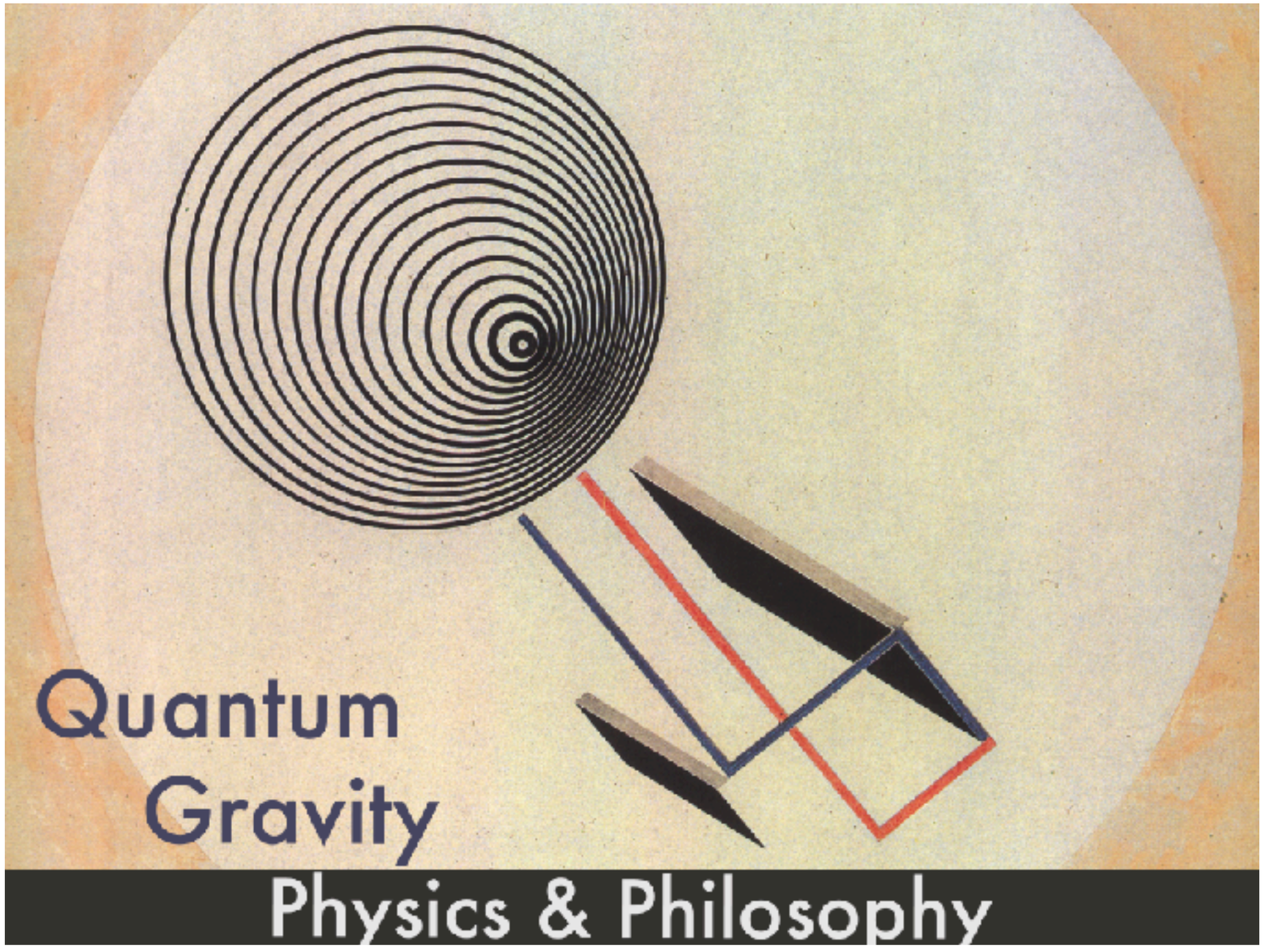




Strominger + Vafa '96

exhibited the first microscopic model of (near extremal, 3-charge 5d) **BH** that reproduced the BH formula

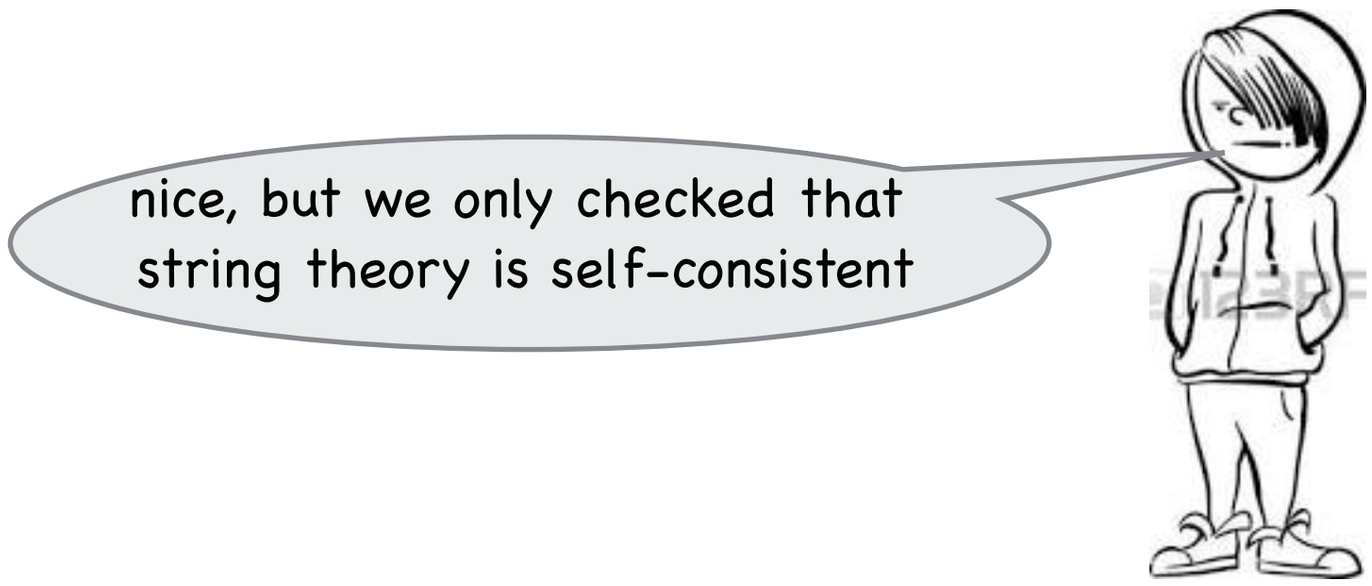
$$\log \mathcal{N} \simeq 2\pi\sqrt{NN_1N_5} = S_{BH}$$



an artist's view

Depending on one ‘s mood:

or:



Are other properties of this BH also described by YM theory?

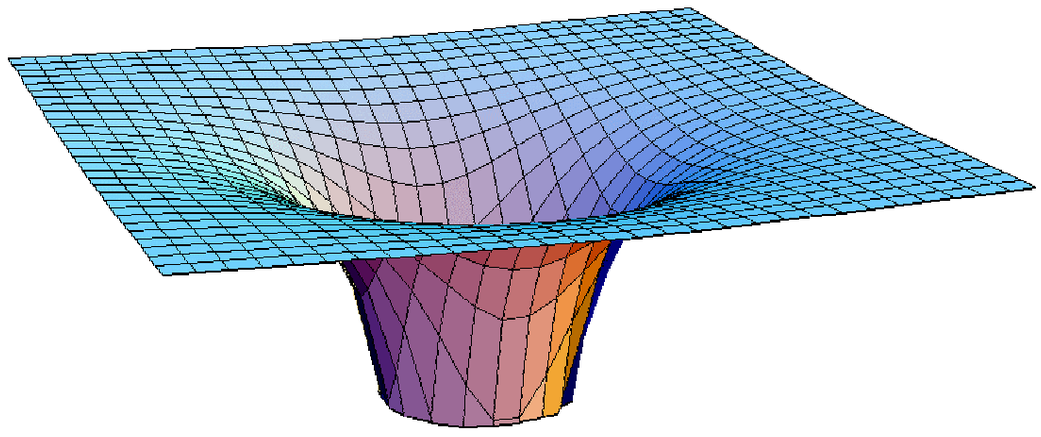
Here enters the crucial insight of Maldacena:

In the D-brane background consider the limit $M_{\rm string},\ M_{\rm Planck} \to \infty$

If the YM theory can be defined autonomously (UV complete) one finds:

YM$_d$ theory $\oplus$ **free gravity waves**

Take same limit in **BH** background.



light because of huge redshift

If the near-horizon region is a `narrow throat' $(\mathrm{AdS}_{d+1} \times M^{9-d})$ one expects:

Closed-string theory in n.h.r.

$\oplus$

free gravity waves

This leads (in simplest setup) to a mathematically-sharp conjecture of **holographic duality**:

$$\mathcal{N}=4 \text{ super YM in } d=4$$

$$\Updownarrow$$

$$\text{type IIB string theory in } \ \mathrm{AdS}_5\times \mathrm{S}^5$$

or for short (and more generally): $\qquad \mathrm{CFT}_d \iff \mathrm{AdS}_{d+1}$

3. Much ado about what ?

So we have a proposal of duality between a `conventional' gauge theory (similar to the SM) and string theory in AdS

In principle they both describe the same physical object, so duality is a statement of mathematical equivalence.

In practice we are limited by the available computational tools:

perturbation theory around classical solutions + a little more

String theory is indeed not defined in any other way, and for a purist, neither is YM (cf Clay millenium problem)

String theory has two free parameters:

string tension: $\frac{1}{\ell_s}^2$ **string coupling:** $g_s = (2\pi)^{\frac{7}{2}} (\frac{\ell_{\rm Planck}}{\ell_s})^4$

in AdS background of radius L the convenient expansion parameters are the two dimensionless ratios

$$(\frac{L}{\ell_s})^4 := \lambda \qquad \text{and} \qquad (\frac{L}{\ell_{\rm Planck}})^4 \sim \frac{\lambda}{g_s}$$

The limit of classical gravity is $L \gg \ell_s,\ \ell_{\rm Planck}$

Finite λ can be sometimes handled exactly via 2d **integrability**,
but there is no non-perturbative definition of the theory for finite g_s

YM theory also has two parameters:

gauge coupling: $g_{\rm YM}$ **# colors**: N_c

The diagrammatic expansion (in powers of $g_{\rm YM}$) can be organized conveniently in terms of $1/N_c$ and the **'t Hooft coupling**

$$\lambda := g_{\rm YM}^2 N_c$$

The planar limit $N_c \to \infty$ simplifies & makes convergent the expansion, but explicit computations are still hard.

Contrary to string theory, there exists however an **action principle**, so background-independent computations can be envisaged (e.g. lattice)

AdS/CFT identifies the parameters λ , as well as $N_c \equiv \frac{\lambda}{4\pi g_s} \sim (\frac{L}{\ell_{\rm Planck}})^4$

The two sides are a priori tractable in opposite regions, but in the planar limit

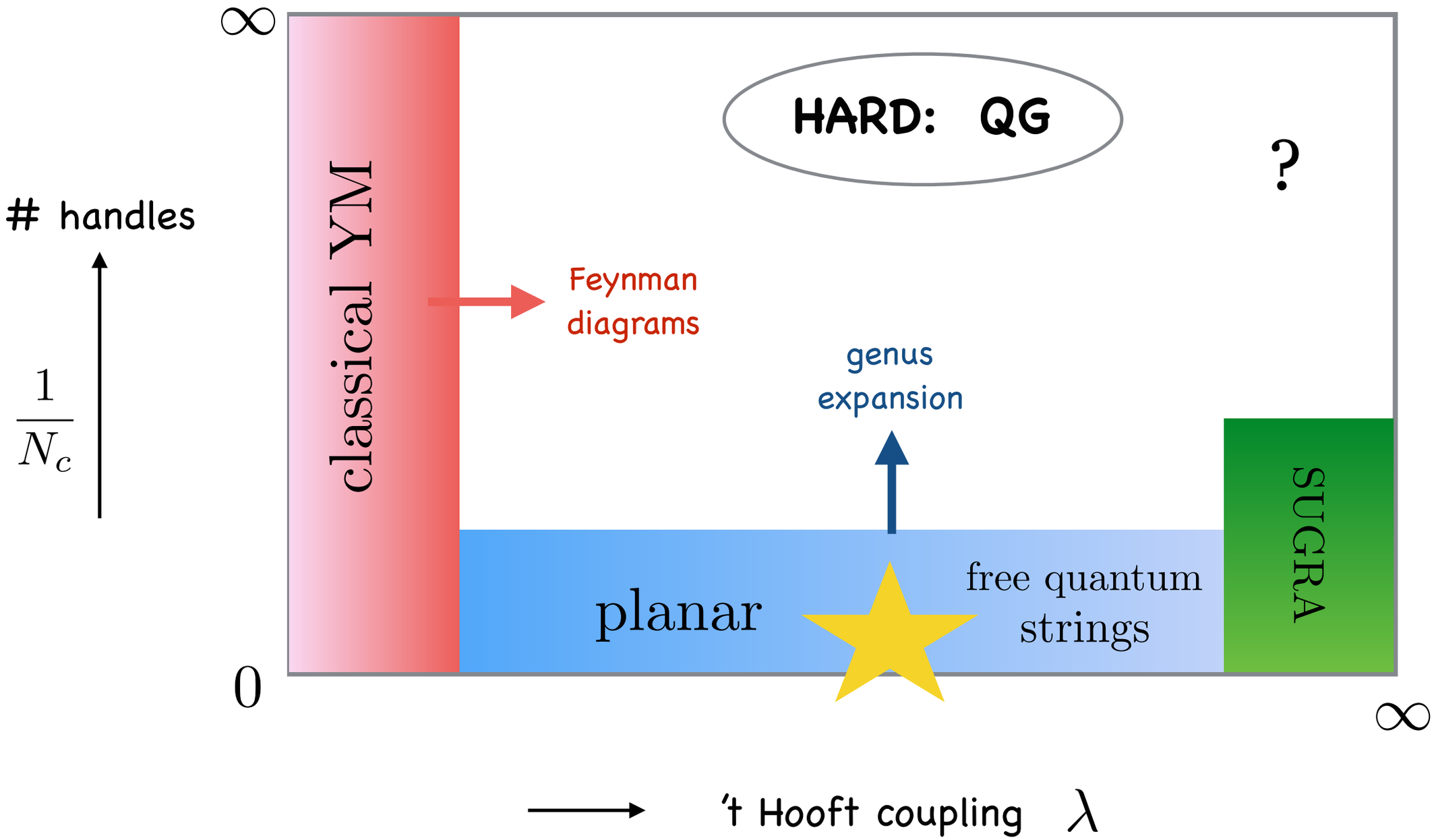
$$N_c \to \infty \qquad \lambda \text{ fixed}$$

planar YM $\Longleftrightarrow$ free string

they could be exactly matched. ★

This required the development of powerful new techniques
of quantum integrability

Minahan+ Zarembo '02; Staudacher, Beisert
N. Gromov, Kazakov, Vieira; . . .


∞
handles
$\frac{1}{N_c}$
classical YM
HARD: QG
?
Feynman diagrams
genus expansion
planar
free quantum strings
SUGRA
0
∞
't Hooft coupling λ

It is also the hallmark of good science to provide solutions
to older, previously unrelated problems.

Computing and resumming Feynman diagrams in 4d YM theory
is extremely tedious, and of great practical importance
(for QCD backgrounds at LHC).

Efforts to solve the planar limit date back to the 70's
(Master field; Eguchi-Kawai reduction; loop equations)
Now, at least in one special theory, it is solved.

An illustrative example: scaling dimension of the (spin=twist=2)
Konishi operator in $\mathcal{N}=4$ sYM

Konishi operator from quantum spectral curve:

Marbeu,Volin arXiv:1411.4758

$g = \sqrt{\lambda}/4\pi$

130 000 Feynman graphs !

$$
\begin{aligned}
\Delta \;=\;& 4 + 12g^2 - 48g^4 + 336g^6 + g^8\big(-2496 + 576\,\zeta_3 - 1440\,\zeta_5\big) \\
&+ g^{10}\big(15168 + 6912\,\zeta_3 - 5184\,\zeta_3^2 - 8640\,\zeta_5 + 30240\,\zeta_7\big) \\
&+ g^{12}\big(-7680 - 262656\,\zeta_3 - 20736\,\zeta_3^2 + 112320\,\zeta_5 + 155520\,\zeta_3\,\zeta_5 + 75600\,\zeta_7 - 489888\,\zeta_9\big) \\
&+ g^{14}\big(-2135040 + 5230080\,\zeta_3 - 421632\,\zeta_3^2 + 124416\,\zeta_3^3 - 229248\,\zeta_5 + 411264\,\zeta_3\,\zeta_5 \\
&\quad - 993600\,\zeta_5^2 - 1254960\,\zeta_7 - 1935360\,\zeta_3\,\zeta_7 - 835488\,\zeta_9 + 7318080\,\zeta_{11}\big) \\
&+ g^{16}\Big(54408192 - 83496960\,\zeta_3 + 7934976\,\zeta_3^2 + 1990656\,\zeta_3^3 - 19678464\,\zeta_5 - 4354560\,\zeta_3\,\zeta_5 \\
&\quad - 3255552\,\zeta_3^2\,\zeta_5 + 2384640\,\zeta_5^2 + 21868704\,\zeta_7 - 6229440\,\zeta_3\,\zeta_7 + 22256640\,\zeta_5\,\zeta_7 \\
&\quad + 9327744\,\zeta_9 + 23224320\,\zeta_3\,\zeta_9 + \frac{65929248}{5}\,\zeta_{11} - 106007616\,\zeta_{13} - \frac{684288}{5}\,Z_{11}^{(2)}\Big) \\
&+ g^{18}\Big(-1014549504 + 1140922368\,\zeta_3 - 51259392\,\zeta_3^2 - 20155392\,\zeta_3^3 + 575354880\,\zeta_5 \\
&\quad - 14294016\,\zeta_3\,\zeta_5 - 26044416\,\zeta_3^2\,\zeta_5 + 55296000\,\zeta_5^2 + 15759360\,\zeta_3\,\zeta_5^2 - 223122816\,\zeta_7 \\
&\quad + 34020864\,\zeta_3\,\zeta_7 + 22063104\,\zeta_3^2\,\zeta_7 - 92539584\,\zeta_5\,\zeta_7 - 113690304\,\zeta_7^2 - 247093632\,\zeta_9 \\
&\quad + 119470464\,\zeta_3\,\zeta_9 - 245099520\,\zeta_5\,\zeta_9 - \frac{186204096}{5}\,\zeta_{11} - 278505216\,\zeta_3\,\zeta_{11} - 253865664\,\zeta_{13} \\
&\quad + 1517836320\,\zeta_{15} + \frac{15676416}{5}\,Z_{11}^{(2)} - 1306368\,Z_{13}^{(2)} + 1306368\,Z_{13}^{(3)}\Big) \\
&+ g^{20}\Big(16445313024 - 13069615104\,\zeta_3 - 1509027840\,\zeta_3^2 + 578949120\,\zeta_3^3 \\
&\quad - 14929920\,\zeta_3^4 - 11247547392\,\zeta_5 + 1213581312\,\zeta_3\,\zeta_5 + 1234206720\,\zeta_3^2\,\zeta_5 \\
&\quad - 70170624\,\zeta_3^3\,\zeta_5 - 1390279680\,\zeta_5^2 - 654842880\,\zeta_3\,\zeta_5^2 + \frac{6966252288}{175}\,\zeta_5^3 \\
&\quad + 377212032\,\zeta_7 - 1610841600\,\zeta_3\,\zeta_7 + 154680192\,\zeta_3^2\,\zeta_7 + 222341760\,\zeta_5\,\zeta_7 \\
&\quad + 133788672\,\zeta_3\,\zeta_5\,\zeta_7 + 868662144\,\zeta_7^2 + 4915257984\,\zeta_9 - 332646912\,\zeta_3\,\zeta_9 \\
&\quad - 91072512\,\zeta_3^2\,\zeta_9 + 1099699200\,\zeta_5\,\zeta_9 + 2275620480\,\zeta_7\,\zeta_9 + \frac{9793211904}{5}\,\zeta_{11} \\
&\quad - 2334572928\,\zeta_3\,\zeta_{11} + 2713772160\,\zeta_5\,\zeta_{11} - \frac{787483944}{175}\,\zeta_{13} + 3372969600\,\zeta_3\,\zeta_{13} \\
&\quad - \frac{4308536566944}{875}\,\zeta_{15} - 21661960320\,\zeta_{17} + \frac{752219136}{5}\,Z_{11}^{(2)} - \frac{5070791808}{175}\,Z_{13}^{(2)} \\
&\quad - \frac{7159104}{7}\,Z_{13}^{(3)} + \frac{2716063488}{175}\,Z_{15}^{(2)} - \frac{17895168}{25}\,Z_{15}^{(3)} + 11943936\,\zeta_3\,Z_{11}^{(2)}\Big) + \mathcal{O}(g^{22})\,, \qquad (85)
\end{aligned}
$$

where $Z_a^{(n)}$ denote single-valued MZV's written in the basis [63]

There exist many problems of QFT at strong coupling

for which only numerical approaches were available, e.g.

Quark-gluon plasma

Quantum critical points (high Tc supra ?)

AdS/CFT provided a new semi-analytic handle to such problems

But we are here interested in the opposite arrow:

Instead of using Einstein eqs. to solve strongly-coupled QFTs,

can we use QFTs to learn about strongly-quantum gravity ?

4. Back to Quantum Gravity

The first thing to observe is that AdS/CFT proposes a (partially) background independent formulation of QG

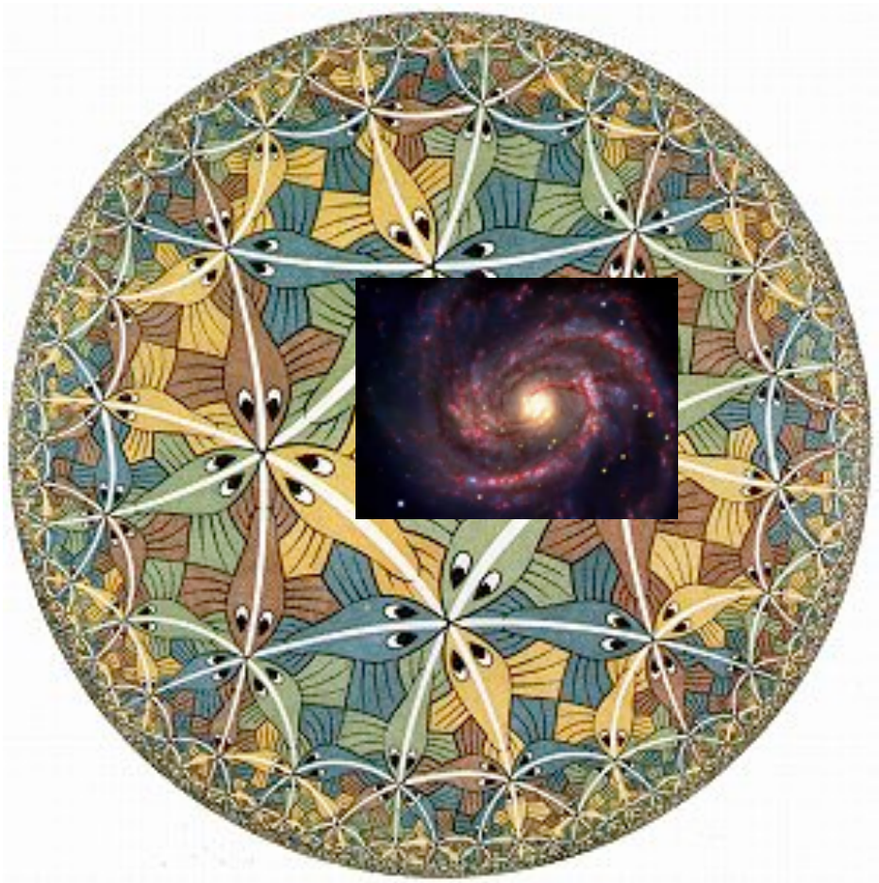
QG with asymptotically AdS boundary conditions is described by $\mathcal{N} = 4$ sYM theory

This is a conventional QFT and, although it is hard to come up with rigorous proof, most physicists have no doubt that it must be free of pathologies:

- No **loss of information**

- No physical **singularities**

M.C. Escher, *Circle Limit III*, 1959.
strictly-speaking this is EAdS2



AdS boundary conditions make a gravitational trap

AdS metric

$$ds^2 = d\rho^2 + e^{2\rho}(-dt^2 + d\vec{x}d\vec{x})$$

At the boundary ($\rho \to \infty$) frequencies suffer infinite blueshift

But in the interior anything can go :

(Small) black holes form and evaporate, singularities appear in the geometric limit etc; It must all be `described' by YM

The issue is summarized by the diverging accounts of the same trip by an (otherwise happily married) couple, Alice and Bob.

Alice falls into a BH and is well-versed in General Relativity, while Bob waits for her outside and is highly quantum.

GR

Alice reaches the horizon after finite time by her clock,
then crosses it sipping coffee (careful only not to be torn apart by tidal forces);

She continues her travel until she hits a spacelike singularity,
where all hell breaks loose (including GR) and time ends.

YM

Bob sees no drama. His wife takes an infinite time to fall in,
but eventually comes out after the evaporation of the BH.

All info on her agenda is intact, though collecting it (and her)
may require detectors at the four corners of the Universe,
and a very expensive machine to put it all back together.

Can these two accounts be reconciled ?

The contradiction may well be a **red herring**,
due to the **`illusions'** of classical intuition (limits)

We know other such paradoxes:

the twin paradox of Special Relativity,

or Schrödinger's cat in QM.

These are paradoxes of the limits $c \to 0$ $\hbar \to \infty$

Likewise, Alice's conflicting account of the trip might be a nightmare dissipated at finite N_c and λ .

AdS/CFT provides a well-motivated (containing GR) & controlable in principle, quantum YM theory for resolving the conflict.

One expects a standard S-matrix :

$$|\mathbf{BH} + \text{Alice} + \text{purse}\rangle \;\rightarrow\; |\mathbf{BH} + \text{Alice} + \text{shopping}\rangle$$

But the technical details of how the `horizon and singularity illusions' arise look awefully hard. Need ingenuity, patience (cf simpler question of confinement) and perhaps a simpler, sharp question to focus the energies.

This is a hot present-day topic, with many interesting (some mutually conflicting) ideas. Let me mention some:

Fuzzballs: horizon and singularity are not unavoidable even in the classical geometric limit
(smooth `fingered' geometries where space ends)

Mathur, . . .

Firewalls: GR breaks down and all hell breaks loose at a BH horizon (Alice and her purse get blown up, much before hitting the singularity)

Almheiri et al

revolutionary: goes against simple application of EP

State dependence: simple observables of an infalling observer depend on the precise quantum state of the BH

Papadodimas + Raju

measurement process not unitary ?

Ryu + Takayanagi

Entanglement & Geometry: geometric limit of quantum entanglement ?

An extension of the BH formula; ER=EPR ?

Quantum chaos: BHs scramble information at a maximal rate;

Rigorous bound on growth of chaos: $\lambda_L \leq 2\pi k_B T/\hbar$

new to specialists

Maldacena+ Shenker + Stanford

Saturating the bound: a guide to models of Schwarzchild horizons

Not everything flies:

e.g. **dS/CFT** did not leave us wiser (up to now)

On going debate. What to hope ?

Ultimately, one would like to address the major observational puzzles:

dark energy; CMB from Big Bang; 5th force(s)

String theory had some empirical successes

unified forces with $\log_{10}(M_{\rm GUT}/{\rm GeV}) \sim 16$

and (is sufficiently developped to suffer) empirical stress

susy breaking; moduli; vacuum stability

Time will show if it is the right/false route to QG, but . . .

. . . the example of QCD has shown that vacuum properties
can be the hardest to compute, even in simple QFTs :

quark bag
models

Yang-Mills 

Will **AdS/CFT** help in resolving these observational puzzles ?

No clear indication or ideas so far.
But here is a quote of a master :

(**PAM Dirac**, Lecture 1 on Quantum Mechanics, Christchurch-New Zealand 1975)

` Hamilton 100 years earlier had set up another form of dynamics …. He pursued this line of investigation just because it led to greater <u>beauty and symmetry</u> of the equations ….

I believe this shows the genius of Hamilton that he was able to follow through a line of work whose importance was not evident until 100 years later.

I learned this (Hamilton 's formulation of mechanics) without at the time realizing whether it would be important or not,
but simply because it was related to things that <u>were</u> important `

Concluding Remarks

- Of various QG proposals, string theory is the most conservative
(gives up no basic principles of QM; has smooth geometric limit)

- Holographic duality comes out of it (almost) as a logical inference,
and extends (but cannot be disconnected from) it.
Dual QGs **are** string theories

- We have a model in which to study BH `paradoxes';
pursue this (& its many spin offs)
& hope it leads to experiment as well